LINEAR FUNCTIONS

1.1 Slopes and Equations of Lines

Your Turn 1

Find the slope of the line through $(1, 5)$ and $(4, 6)$.

Let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (4, 6)$.

$$m = \frac{6 - 5}{4 - 1} = \frac{1}{3}$$

Your Turn 2

Find the equation of the line with x-intercept -4 and y-intercept 6.

We know that $b = 6$ and that the line crosses the axes at $(-4, 0)$ and $(0, 6)$. Use these two intercepts to find the slope m.

$$m = \frac{6 - 0}{0 - (-4)} = \frac{6}{4} = \frac{3}{2}$$

Thus the equation for the line in slope-intercept form is $y = \frac{3}{2}x + 6$.

Your Turn 3

Find the slope of the line whose equation is $8x + 3y = 5$.

Solve the equation for y.

$$8x + 3y = 5$$
$$3y = -8x + 5$$
$$y = -\frac{8}{3}x + \frac{5}{3}$$

The slope is $-8/3$.

Your Turn 4

Find the equation (in slope-intercept form) of the line through $(2, 9)$ and $(5, 3)$.

First find the slope.

$$m = \frac{3 - 9}{5 - 2} = \frac{-6}{3} = -2$$

Now use the point-slope form, with $(x_1, y_1) = (5, 3)$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -2(x - 5)$$
$$y - 3 = -2x + 10$$
$$y = -2x + 13$$

Your Turn 5

Find (in slope-intercept form) the equation of the line that passes through the point $(4, 5)$ and is parallel to the line $3x - 6y = 7$.

First find the slope of the line $3x - 6y = 7$ by solving this equation for y.

$$3x - 6y = 7$$
$$6y = 3x - 7$$
$$y = \frac{3}{6}x - \frac{7}{6}$$
$$y = \frac{1}{2}x - \frac{7}{6}$$

Since the line we are to find is parallel to this line, it will also have slope $1/2$. Use the point-slope form with $(x_1, y_1) = (4, 5)$.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{1}{2}(x - 4)$$
$$y - 5 = \frac{1}{2}x - 2$$
$$y = \frac{1}{2}x + 3$$

Your Turn 6

Find (in slope-intercept form) the equation of the line that passes through the point $(3, 2)$ and is perpendicular to the line $2x + 3y = 4$.

First find the slope of the line $2x + 3y = 4$ by solving this equation for y.

$$2x + 3y = 4$$
$$3y = -2x + 4$$
$$y = -\frac{2}{3}x + \frac{4}{3}$$

Since the line we are to find is perpendicular to a line with slope $-2/3$, , it will have slope $3/2$. (Note that $(-2/3)(3/2) = -1$.)

Use the point-slope form with $(x_1, y_1) = (3, 2)$.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{3}{2}(x - 3)$$
$$y - 2 = \frac{3}{2}x - \frac{9}{2}$$
$$y = \frac{3}{2}x - \frac{5}{2}$$

1.1 Warmup Exercises

W1. $\dfrac{15 - (-3)}{-2 - 4} = \dfrac{18}{-6} = -3$

W2. $y - (-3) = -2(x + 5)$
$$y + 3 = -2x - 10$$
$$y = -2x - 13$$

W3. $y - \dfrac{1}{2} = \dfrac{2}{5}\left(x + \dfrac{1}{3}\right)$
$$y - \frac{1}{2} = \frac{2}{5}x + \frac{2}{15}$$
$$y = \frac{2}{5}x + \frac{4}{30} + \frac{15}{30}$$
$$y = \frac{2}{5}x + \frac{19}{30}$$

W4. $2x - 3y = 7$
$$-3y = -2x + 7$$
$$y = \frac{2}{3}x - \frac{7}{3}$$

1.1 Exercises

1. Find the slope of the line through $(4, 5)$ and $(-1, 2)$.
$$m = \frac{5 - 2}{4 - (-1)}$$
$$= \frac{3}{5}$$

2. Find the slope of the line through $(5, -4)$ and $(1, 3)$.
$$m = \frac{3 - (-4)}{1 - 5}$$
$$= \frac{3 + 4}{-4}$$
$$= -\frac{7}{4}$$

3. Find the slope of the line through $(8, 4)$ and $(8, -7)$.
$$m = \frac{4 - (-7)}{8 - 8} = \frac{11}{0}$$
The slope is undefined; the line is vertical.

4. Find the slope of the line through $(1, 5)$ and $(-2, 5)$.
$$m = \frac{5 - 5}{-2 - 1} = \frac{0}{-3} = 0$$

5. $y = x$

Using the slope-intercept form, $y = mx + b$, we see that the slope is 1.

6. $y = 3x - 2$

This equation is in slope-intercept form, $y = mx + b$. Thus, the coefficient of the x-term, 3, is the slope.

7. $5x - 9y = 11$

Rewrite the equation in slope-intercept form.
$$9y = 5x - 11$$
$$y = \frac{5}{9}x - \frac{11}{9}$$

The slope is $\frac{5}{9}$.

8. $4x + 7y = 1$

Rewrite the equation in slope-intercept form.
$$7y = 1 - 4x$$
$$\frac{1}{7}(7y) = \frac{1}{7}(1) - \frac{1}{7}(4x)$$
$$y = \frac{1}{7} - \frac{4}{7}x$$
$$y = -\frac{4}{7}x + \frac{1}{7}$$

The slope is $-\frac{4}{7}$.

9. $x = 5$

This is a vertical line. The slope is undefined.

10. The x-axis is the horizontal line $y = 0$.
Horizontal lines have a slope of 0.

11. $y = 8$

This is a horizontal line, which has a slope of 0.

12. $y = -6$

By rewriting this equation in the slope-intercept form, $y = mx + b$, we get $y = 0x - 6$, with the slope, m, being 0.

13. Find the slope of a line parallel to $6x - 3y = 12$.

Rewrite the equation in slope-intercept form.
$$-3y = -6x + 12$$
$$y = 2x - 4$$

The slope is 2, so a parallel line will also have slope 2.

14. Find the slope of a line perpendicular to $8x = 2y - 5$.

First, rewrite the given equation in slope-intercept form.

$$8x = 2y - 5$$
$$8x + 5 = 2y$$
$$4x + \frac{5}{2} = y \quad \text{or} \quad y = 4x + \frac{5}{2}$$

Let m be the slope of any line perpendicular to the given line. Then.

$$4 \cdot m = -1$$
$$m = -\frac{1}{4}.$$

15. The line goes through $(1, 3)$, with slope $m = -2$. Use point-slope form.

$$y - 3 = -2(x - 1)$$
$$y = -2x + 2 + 3$$
$$y = -2x + 5$$

16. The line goes through $(2, 4)$, with slope $m = -1$. Use point-slope form.

$$y - 4 = -1(x - 2)$$
$$y - 4 = -x + 2$$
$$y = -x + 6$$

17. The line goes through $(-5, -7)$ with slope $m = 0$. Use point-slope form.

$$y - (-7) = 0[x - (-5)]$$
$$y + 7 = 0$$
$$y = -7$$

18. The line goes through $(-8, 1)$, with undefined slope. Since the slope is undefined, the line is vertical. The equation of the vertical line passing through $(-8, 1)$ is $x = -8$.

19. The line goes through $(4, 2)$ and $(1, 3)$. Find the slope, then use point-slope form with either of the two given points.

$$m = \frac{3 - 2}{1 - 4} = -\frac{1}{3}$$
$$y - 3 = -\frac{1}{3}(x - 1)$$
$$y = -\frac{1}{3}x + \frac{1}{3} + 3$$
$$y = -\frac{1}{3}x + \frac{10}{3}$$

20. The line goes through $(8, -1)$ and $(4, 3)$. Find the slope, then use point-slope form with either of the two given points.

$$m = \frac{3 - (-1)}{4 - 8}$$
$$= \frac{3 + 1}{-4}$$
$$= \frac{4}{-4} = -1$$
$$y - (-1) = -1(x - 8)$$
$$y + 1 = -x + 8$$
$$y = -x + 7$$

21. The line goes through $\left(\frac{2}{3}, \frac{1}{2}\right)$ and $\left(\frac{1}{4}, -2\right)$.

$$m = \frac{-2 - \frac{1}{2}}{\frac{1}{4} - \frac{2}{3}} = \frac{-\frac{4}{2} - \frac{1}{2}}{\frac{3}{12} - \frac{8}{12}}$$
$$m = \frac{-\frac{5}{2}}{-\frac{5}{12}} = \frac{60}{10} = 6$$
$$y - (-2) = 6\left(x - \frac{1}{4}\right)$$
$$y + 2 = 6x - \frac{3}{2}$$
$$y = 6x - \frac{3}{2} - 2$$
$$y = 6x - \frac{3}{2} - \frac{4}{2}$$
$$y = 6x - \frac{7}{2}$$

22. The line goes through $\left(-2, \frac{3}{4}\right)$ and $\left(\frac{2}{3}, \frac{5}{2}\right)$.

$$m = \frac{\frac{5}{2} - \frac{3}{4}}{\frac{2}{3} - (-2)} = \frac{\frac{10}{4} - \frac{3}{4}}{\frac{2}{3} + \frac{6}{3}}$$
$$= \frac{\frac{7}{4}}{\frac{8}{3}} = \frac{21}{32}$$
$$y - \frac{3}{4} = \frac{21}{32}[x - (-2)]$$
$$y - \frac{3}{4} = \frac{21}{32}x + \frac{42}{32}$$
$$y = \frac{21}{32}x + \frac{42}{32} + \frac{3}{4}$$
$$y = \frac{21}{32}x + \frac{21}{16} + \frac{12}{16}$$
$$y = \frac{21}{32}x + \frac{33}{16}$$

23. The line goes through $(-8, 4)$ and $(-8, 6)$.
$$m = \frac{4 - 6}{-8 - (-8)} = \frac{-2}{0};$$
which is undefined.
This is a vertical line; the value of x is always -8.
The equation of this line is $x = -8$.

24. The line goes through $(-1, 3)$ and $(0, 3)$.
$$m = \frac{3 - 3}{-1 - 0} = \frac{0}{-1} = 0$$
This is a horizontal line; the value of y is always 3.
The equation of this line is $y = 3$.

25. The line has x-intercept -6 and y-intercept -3.
Two points on the line are $(-6, 0)$ and $(0, -3)$.
Find the slope; then use slope-intercept form.
$$m = \frac{-3 - 0}{0 - (-6)} = \frac{-3}{6} = -\frac{1}{2}$$
$$b = -3$$
$$y = -\frac{1}{2}x - 3$$
$$2y = -x - 6$$
$$x + 2y = -6$$

26. The line has x-intercept -2 and y-intercept 4.
Two points on the line are $(-2, 0)$ and $(0, 4)$. Find the slope; then use slope-intercept form.
$$m = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$$
$$y = mx + b$$
$$y = 2x + 4$$
$$2x - y = -4$$

27. The vertical line through $(-6, 5)$ goes through the point $(-6, 0)$, so the equation is $x = -6$.

28. The line is horizontal, through $(8, 7)$.
The line has an equation of the form $y = k$ where k is the y-coordinate of the point. In this case, $k = 7$, so the equation is $y = 7$.

29. Write an equation of the line through $(-4, 6)$, parallel to $3x + 2y = 13$.

Rewrite the equation of the given line in slope-intercept form.
$$3x + 2y = 13$$
$$2y = -3x + 13$$
$$y = -\frac{3}{2}x + \frac{13}{2}$$

The slope is $-\frac{3}{2}$.

Use $m = -\frac{3}{2}$ and the point $(-4, 6)$ in the point-slope form.
$$y - 6 = -\frac{3}{2}[x - (-4)]$$
$$y = -\frac{3}{2}(x + 4) + 6$$
$$y = -\frac{3}{2}x - 6 + 6$$
$$y = -\frac{3}{2}x$$
$$2y = -3x$$
$$3x + 2y = 0$$

30. Write the equation of the line through $(2, -5)$, parallel to $y - 4 = 2x$. Rewrite the equation in slope-intercept form.
$$y - 4 = 2x$$
$$y = 2x + 4$$
The slope of this line is 2.
Use $m = 2$ and the point $(2, -5)$ in the point-slope form.
$$y - (-5) = 2(x - 2)$$
$$y + 5 = 2x - 4$$
$$y = 2x - 9$$
$$2x - y = 9$$

31. Write an equation of the line through $(3, -4)$, perpendicular to $x + y = 4$.

Rewrite the equation of the given line as
$$y = -x + 4.$$
The slope of this line is -1. To find the slope of a perpendicular line, solve
$$-1m = -1.$$
$$m = 1$$
Use $m = 1$ and $(3, -4)$ in the point-slope form.
$$y - (-4) = 1(x - 3)$$
$$y = x - 3 - 4$$
$$y = x - 7$$
$$x - y = 7$$

32. Write the equation of the line through $(-2, 6)$, perpendicular to $2x - 3y = 5$.

Rewrite the equation in slope-intercept form.
$$2x - 3y = 5$$
$$-3y = -2x + 5$$
$$y = \frac{2}{3}x - \frac{5}{3}$$

The slope of this line is $\frac{2}{3}$. To find the slope of a perpendicular line, solve

$$\frac{2}{3}m = -1.$$

$$m = -\frac{3}{2}$$

Use $m = -\frac{3}{2}$ and $(-2, 6)$ in the point-slope form.

$$y - 6 = -\frac{3}{2}[x - (-2)]$$

$$y - 6 = -\frac{3}{2}(x + 2)$$

$$y - 6 = -\frac{3}{2}x - 3$$

$$y = -\frac{3}{2}x + 3$$

$$2y = -3x + 6$$

$$3x + 2y = 6$$

33. Write an equation of the line with y-intercept 4, perpendicular to $x + 5y = 7$.

Find the slope of the given line.

$$x + 5y = 7$$

$$5y = -x + 7$$

$$y = -\frac{1}{5}x + \frac{7}{5}$$

The slope is $-\frac{1}{5}$, so the slope of the perpendicular line will be 5. If the y-intercept is 4, then using the slope-intercept form we have

$$y = mx + b$$

$$y = 5x + 4, \quad \text{or} \quad 5x - y = -4$$

34. Write the equation of the line with x-intercept $-\frac{2}{3}$, perpendicular to $2x - y = 4$.

Find the slope of the given line.

$$2x - y = 4$$

$$2x - 4 = y$$

The slope of this line is 2. Since the lines are perpendicular, the slope of the needed line is $-\frac{1}{2}$.

The line also has an x-intercept of $-\frac{2}{3}$. Thus, it passes through the point $\left(-\frac{2}{3}, 0\right)$.

Using the point-slope form, we have

$$y - 0 = -\frac{1}{2}\left[x - \left(-\frac{2}{3}\right)\right]$$

$$y = -\frac{1}{2}\left(x + \frac{2}{3}\right)$$

$$y = -\frac{1}{2}x - \frac{1}{3}$$

$$6y = -3x - 2$$

$$3x + 6y = -2$$

35. Do the points $(4, 3)$, $(2, 0)$, and $(-18, -12)$ lie on the same line?

Find the slope between $(4, 3)$ and $(2, 0)$.

$$m = \frac{0 - 3}{2 - 4} = \frac{-3}{-2} = \frac{3}{2}$$

Find the slope between $(4, 3)$ and $(-18, -12)$.

$$m = \frac{-12 - 3}{-18 - 4} = \frac{-15}{-22} = \frac{15}{22}$$

Since these slopes are not the same, the points do not lie on the same line.

36. (a) Write the given line in slope-intercept form.

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

This line has a slope of $-\frac{2}{3}$. The desired line has a slope of $-\frac{2}{3}$ since it is parallel to the given line. Use the definition of slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{2}{3} = \frac{2 - (-1)}{k - 4}$$

$$-\frac{2}{3} = \frac{3}{k - 4}$$

$$-2(k - 4) = (3)(3)$$

$$-2k + 8 = 9$$

$$-2k = 1$$

$$k = -\frac{1}{2}$$

(b) Write the given line in slope-intercept form.

$$5x - 2y = -1$$

$$2y = 5x + 1$$

$$y = \frac{5}{2}x + \frac{1}{2}$$

This line has a slope of $\frac{5}{2}$. The desired line has a slope of $-\frac{2}{5}$ since it is perpendicular to the given line. Use the definition of slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - (-1)}{k - 4}$$

$$-\frac{2}{5} = \frac{2 + 1}{k - 4}$$

$$\frac{-2}{5} = \frac{3}{k - 4}$$

$$-2(k - 4) = (3)(5)$$

$$-2k + 8 = 15$$

$$-2k = 7$$

$$k = -\frac{7}{2}$$

37. A parallelogram has 4 sides, with opposite sides parallel. The slope of the line through $(1, 3)$ and $(2, 1)$ is

$$m = \frac{3 - 1}{1 - 2}$$

$$= \frac{2}{-1}$$

$$= -2.$$

The slope of the line through $\left(-\frac{5}{2}, 2\right)$ and $\left(-\frac{7}{2}, 4\right)$

is

$$m = \frac{2 - 4}{-\frac{5}{2} - \left(-\frac{7}{2}\right)} = \frac{-2}{1} = -2.$$

Since these slopes are equal, these two sides are parallel.

The slope of the line through $\left(-\frac{7}{2}, 4\right)$ and $(1, 3)$ is

$$m = \frac{4 - 3}{-\frac{7}{2} - 1} = \frac{1}{-\frac{9}{2}} = -\frac{2}{9}.$$

Slope of the line through $\left(-\frac{5}{2}, 2\right)$ and $(2, 1)$ is

$$m = \frac{2 - 1}{-\frac{5}{2} - 2} = \frac{1}{-\frac{9}{2}} = -\frac{2}{9}.$$

Since these slopes are equal, these two sides are parallel.

Since both pairs of opposite sides are parallel, the quadrilateral is a parallelogram.

38. Two lines are perpendicular if the product of their slopes is -1.

The slope of the diagonal containing $(4, 5)$ and $(-2, -1)$ is

$$m = \frac{5 - (-1)}{4 - (-2)} = \frac{6}{6} = 1.$$

The slope of the diagonal containing $(-2, 5)$ and $(4, -1)$ is

$$m = \frac{5 - (-1)}{-2 - 4} = \frac{6}{-6} = -1.$$

The product of the slopes is $(1)(-1) = -1,$ so the diagonals are perpendicular.

39. The line goes through $(0, 2)$ and $(-2, 0)$

$$m = \frac{2 - 0}{0 - (-2)} = \frac{2}{2} = 1$$

The correct choice is (a).

40. The line goes through $(1, 3)$ and $(2, 0)$.

$$m = \frac{3 - 0}{1 - 2} = \frac{3}{-1} = -3$$

The correct choice is (f).

41. The line appears to go through $(0, 0)$ and $(-1, 4)$.

$$m = \frac{4 - 0}{-1 - 0} = \frac{4}{-1} = -4$$

42. The line goes through $(-2, 0)$ and $(0, 1)$.

$$m = \frac{1 - 0}{0 - (-2)} = \frac{1}{2}$$

43. (a) See the figure in the textbook.

Segment MN is drawn perpendicular to segment PQ. Recall that MQ is the length of segment MQ.

$$m_1 = \frac{\Delta y}{\Delta x} = \frac{MQ}{PQ}$$

From the diagram, we know that $PQ = 1$.

Thus, $m_1 = \frac{MQ}{1},$ so MQ has length m_1.

(b) $m_2 = \frac{\Delta y}{\Delta x} = \frac{-QN}{PQ} = \frac{-QN}{1}$

$$QN = -m_2$$

(c) Triangles MPQ, PNQ, and MNP are right triangles by construction. In triangles MPQ and MNP,

$$\text{angle } M = \text{angle } M,$$

and in the right triangles PNQ and MNP,

$$\text{angle } N = \text{angle } N.$$

Since all right angles are equal, and since triangles with two equal angles are similar, triangle MPQ is similar to triangle MNP and triangle PNQ is similar to triangle MNP.

Therefore, triangles MNQ and PNQ are similar to each other.

(d) Since corresponding sides in similar triangles are proportional,

$$MQ = k \cdot PQ \quad \text{and} \quad PQ = k \cdot QN.$$

$$\frac{MQ}{PQ} = \frac{k \cdot PQ}{k \cdot QN}$$

$$\frac{MQ}{PQ} = \frac{PQ}{QN}$$

From the diagram, we know that $PQ = 1$.

$$MQ = \frac{1}{QN}$$

From (a) and (b), $m_1 = MQ$ and $-m_2 = QN$.

Substituting, we get

$$m_1 = \frac{1}{-m_2}.$$

Multiplying both sides by m_2, we have

$m_1 m_2 = -1$.

44. (a) Multiplying both sides of the equation $\frac{x}{a} + \frac{y}{b} = 1$ by ab, we have

$$ab\left(\frac{x}{a}\right) + ab\left(\frac{y}{b}\right) = ab(1)$$

$$bx + ay = ab.$$

Solve this equation for y.

$$bx + ay = ab$$

$$ay = ab - bx$$

$$y = \frac{ab - bx}{a}$$

$$y = -\frac{b}{a}x + b$$

If we let $m = -\frac{b}{a}$, then the equation becomes

$$y = mx + b.$$

(b) Let $y = 0$.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + 0 = 1$$

$$\frac{x}{a} = 1$$

$$x = a$$

The x-intercept is a.

Let $x = 0$.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$0 + \frac{y}{b} = 1$$
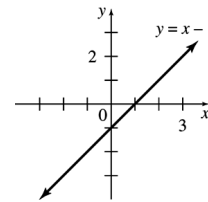
$$\frac{y}{b} = 1$$

$$y = b$$

The y-intercept is b.

(c) If the equation of a line is written as $\frac{x}{a} + \frac{y}{b} = 1$, we immediately know the intercepts of the line, which are a and b.
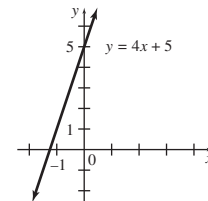
45. $y = x - 1$

Three ordered pairs that satisfy this equation are $(0, -1)$, $(1, 0)$, and $(4, 3)$. Plot these points and draw a line through them.
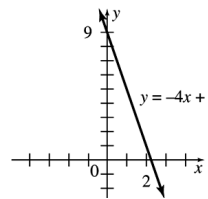

46. $y = 4x + 5$

Three ordered pairs that satisfy this equation are $(-2, -3)$, $(-1, 1)$, and $(0, 5)$. Plot these points and draw a line through them.
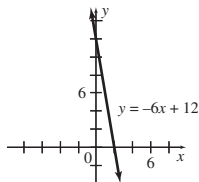

47. $y = -4x + 9$

Three ordered pairs that satisfy this equation are $(0, 9)$, $(1, 5)$, and $(2, 1)$. Plot these points and draw a line through them.

48. $y = -6x + 12$

There ordered pairs that satisfy this equation are $(0, 12)$, $(1, 6)$, and $(2, 0)$. Plot these points and draw a line through them.

49. $2x - 3y = 12$

Find the intercepts.

If $y = 0$, then

$$2x - 3(0) = 12$$
$$2x = 12$$
$$x = 6$$

so the x-intercept is 6.

If $x = 0$, then

$$2(0) - 3y = 12$$
$$-3y = 12$$
$$y = -4$$

so the y-intercept is -4.

Plot the ordered pairs $(6, 0)$ and $(0, -4)$ and draw a line through these points. (A third point may be used as a check.)
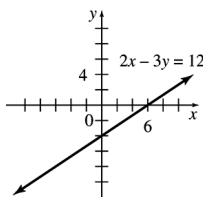
50. $3x - y = -9$
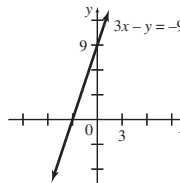
Find the intercepts.

If $y = 0$, then

$$3x - 0 = -9$$
$$3x = -9$$
$$x = -3$$

If $x = 0$, then

$$3(0) - y = -9$$
$$-y = -9$$
$$y = 9$$

so the y-intercept is 9.

Plot the ordered pairs $(-3, 0)$ and $(0, 9)$ and draw a line through these points. (A third point may be used as a check.)

51. $3y - 7x = -21$

Find the intercepts.

If $y = 0$, then

$$3(0) + 7x = -21$$
$$-7x = -21$$
$$x = 3$$

so the x-intercept is 3.

If $x = 0$, then

$$3y - 7(0) = -21$$
$$3y = -21$$
$$y = -7$$

So the y-intercepts is -7.

Plot the ordered pairs $(3, 0)$ and $(0, -7)$ and draw a line through these points. (A third point may be used as a check.)
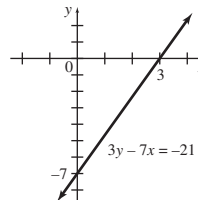
52. $5y + 6x = 11$

Find the intercepts.

If $y = 0$, then

$$5(0) + 6x = 11$$
$$6x = 11$$
$$x = \frac{11}{6}$$

so the x-intercept is $\frac{11}{6}$.

If $x = 0$, then

$$5y + 6(0) = 11$$
$$5y = 11$$
$$y = \frac{11}{5}$$

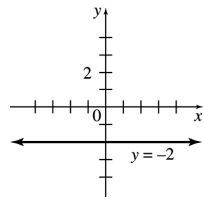so the y-intercept is $\frac{11}{5}$.

Plot the ordered pairs $\left(\frac{11}{6}, 0\right)$ and $\left(0, \frac{11}{5}\right)$ and draw a line through these points. (A third point may be used as a check.)
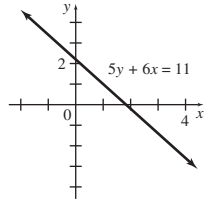

53. $y = -2$

The equation $y = -2$, or, equivalently, $y = 0x - 2$, always gives the same y-value, -2, for any value of x. The graph of this equation is the horizontal line with y-intercept -2.


54. $x = 4$

For any value of y, the x-value is 4. Because all ordered pairs that satisfy this equation have the same first number, this equation does not represent a function. The graph is the vertical line with x-intercept 4.
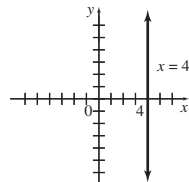

55. $x + 5 = 0$

This equation may be rewritten as $x = -5$. For any value of y, the x-value is -5. Because all ordered pairs that satisfy this equation have the same first number, this equation does not represent a function. The graph is the vertical line with x-intercept -5.
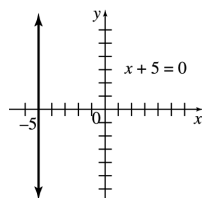
56. $y + 8 = 0$

This equation may be rewritten as $y = -8$, or, equivalently, $y = 0x + -8$. The y-value is -8

for any value of x. The graph is the horizontal line with y-intercept -8.
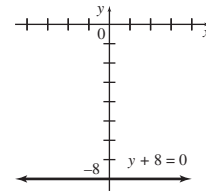
57. $y = 2x$

Three ordered pairs that satisfy this equation are $(0, 0)$, $(-2, -4)$, and $(2, 4)$. Use these points to draw the graph.
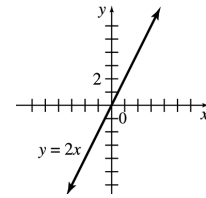
58. $y = -5x$

Three ordered pairs that satisfy this equation are $(0, 0)$, $(-1, 5)$, and $(1, -5)$. Use these points to draw the graph.
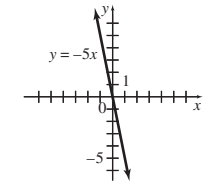
59. $x + 4y = 0$

If $y = 0$, then $x = 0$, so the x-intercept is 0. If $x = 0$, then $y = 0$, so the y-intercept is 0. Both intercepts give the same ordered pair, $(0, 0)$. To get a second point, choose some other value of x (or y). For example if $x = 4$, then

$$x + 4y = 0$$
$$4 + 4y = 0$$
$$4y = -4$$
$$y = -1,$$

giving the ordered pair $(4, -1)$. Graph the line through $(0, 0)$ and $(4, -1)$.
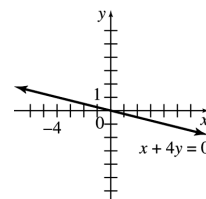
60. $3x - 5y = 0$

If $y = 0$, then $x = 0$, so the x-intercept is 0. If $x = 0$, then $y = 0$, so the y-intercept is 0. Both intercepts give the same ordered pair (0, 0).

To get a second point, choose some other value of x (or y). For example, if $x = 5$, then

$$3x - 5y = 0$$
$$3(5) - 5y = 0$$
$$15 - 5y = 0$$
$$-5y = -15$$
$$y = 3$$

giving the ordered pair (5, 3). Graph the line through (0, 0) and (5, 3).

61. (a) The line goes through (2, 27,000) and (5, 63,000).

$$m = \frac{63,000 - 27,000}{5 - 2}$$
$$= 12,000$$
$$y - 27,000 = 12,000(x - 2)$$
$$y - 27,000 = 12,000x - 24,000$$
$$y = 12,000x + 3000$$

(b) Let $y = 100,000$; find x.

$$100,000 = 12,000x + 3000$$
$$97,000 = 12,000x$$
$$8.08 = x$$

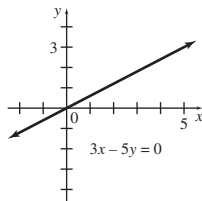Sales would surpass $100,000 after 8 years, 1 month.

62. (a) The line goes through (100, 126) and (120, 144).

$$m = \frac{144 - 126}{120 - 100}$$
$$= 0.9$$

The average cost of producing gourmet cupcakes increases by $0.90 per cupcake.

(b) Use the point slope form with the given points.

$$y - 126 = 0.9(x - 100)$$
$$y = 0.9x - 90 + 126$$
$$y = 0.9x + 36$$

(c) $y = 0.9x + 36$

$$y = 0.9(180) + 36 = 198$$

180 gourmet cupcakes would cost $198.

63. (a)
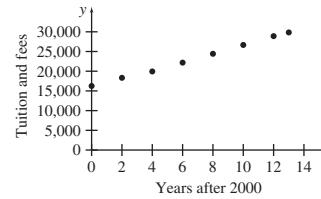
Yes, the data appear to lie roughly along a straight line.

(b) The line goes through $(0, 16,072)$ and $(13, 30,094)$.

$$m = \frac{30,094 - 16,072}{13 - 0} \approx 1078.6$$
$$b = 16,072$$
$$y = 1078.6t + 16,072$$

The slope 1078.6 indicates that tuition and fees have increased approximately $1079 per year.

(c) The year 2025 is too far in the future to rely on this equation to predict costs; too many other factors may influence these costs by then.

64. (a)
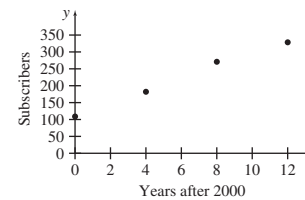
(b) The line goes through $(0, 109.48)$ and $(12, 326.48)$.

$$m = \frac{326.48 - 109.48}{12 - 0}$$
$$\approx 18.083$$
$$b = 109.48$$
$$y = 18.083t + 109.48$$

(c) The line goes through $(4, 182,14)$ and $(12, 326.48)$.

$$m = \frac{326.48 - 182.14}{12 - 4} \approx 18.043$$

Use $(4, 182.14)$ and the point-slope form.

$$y - 182.14 = 18.043(t - 4)$$
$$y = 18.043t - 72.172 + 182.14$$
$$y = 18.043t + 109.97$$

(d) The data is *approximately* linear because all the data points do not fall on a straight line. So the lines between different pairs of points have different slopes that are close in value.

(e) $y = 18.083t + 109.48$
$$y = 18.083(10) + 109.48$$
$$y = 290.31 \text{ million subscribers}$$

$$y = 18.043t + 109.97$$
$$y = 18.043(10) + 109.97$$
$$y = 290.40 \text{ million subscribers}$$

Both the estimated values are slightly less than the actual number of subscribers of 296.29 million.

65. (a) The line goes through $(3, 100)$ and $(32, 229.6)$.

$$m = \frac{229.6 - 100}{32 - 3} \approx 4.469$$

Use the point $(3, 100)$ and the point-slope form.

$$y - 100 = 4.469(t - 3)$$
$$y = 4.469t - 13.407 + 100$$
$$y = 4.469t + 86.593$$

(b) The year 2000 corresponds to $t = 2000 - 1980 = 20$.

$$y = 4.469(20) + 86.593$$
$$y \approx 176.0$$

The predicted value is slightly more than the actual CPI of 172.2.

(c) The annual CPI is increasing at a rate of approximately 4.5 units per year.

66. (a) The line goes through $(4, 0.17)$ and $(7, 0.33)$.

$$m = \frac{0.33 - 0.17}{7 - 4}$$
$$= \frac{0.16}{3} \approx 0.053$$

$$y - 0.33 = \frac{0.16}{3}(t - 7)$$
$$y - 0.33 = 0.053t - 0.373$$
$$y \approx 0.053t - 0.043$$

(b) Let $y = 0.5$; solve for t.

$$0.5 = 0.053t - 0.043$$
$$0.543 = 0.053t$$
$$10.2 = t$$

In about 10.2 years, half of these patients will have AIDS.

67. (a) Let $x =$ age.

$$u = 0.85(220 - x) = 187 - 0.85x$$
$$l = 0.7(200 - x) = 154 - 0.7x$$

(b) $u = 187 - 0.85(20) = 170$
$$l = 154 - 0.7(20) = 140$$

The target heart rate zone is 140 to 170 beats per minute.

(c) $u = 187 - 0.85(40) = 153$
$$l = 154 - 0.7(40) = 126$$

The target heart rate zone is 126 to 153 beats per minute.

(d) $154 - 0.7x = 187 - 0.85(x + 36)$
$$154 - 0.7x = 187 - 0.85x - 30.6$$
$$154 - 0.7x = 156.4 - 0.85x$$
$$0.15x = 2.4$$
$$x = 16$$

The younger woman is 16; the older woman is $16 + 36 = 52$. $l = 0.7(220 - 16) \approx 143$ beats per minute.

68. Let x represent the force and y represent the speed. The linear function contains the points $(0.75, 2)$ and $(0.93, 3)$.

$$m = \frac{3 - 2}{0.93 - 0.75} = \frac{1}{0.18}$$
$$= \frac{1}{\frac{18}{100}} = \frac{100}{18} = \frac{50}{9}$$

Use point-slope form to write the equation.

$$y - 2 = \frac{50}{9}(x - 0.75)$$
$$y - 2 = \frac{50}{9}x - \frac{50}{9}(0.75)$$
$$y = \frac{50}{9}x - \frac{75}{18} + 2$$
$$y = \frac{50}{9}x - \frac{13}{6}$$

Now determine y, the speed, when x, the force, is 1.16.

$$y = \frac{50}{9}(1.16) - \frac{13}{6}$$
$$= \frac{58}{9} - \frac{13}{6} = \frac{77}{18} \approx 4.3$$

The pony switches from a trot to a gallop at approximately 4.3 meters per second.

69. Let $x = 0$ correspond to 1900. Then the "life expectancy from birth" line contains the points $(0, 46)$ and $(110, 78.7)$.

$$m = \frac{78.7 - 46}{110 - 0} = \frac{32.7}{110} \approx 0.297$$

Since $(0, 46)$ is one of the points, the line is given by the equation.

$$y = 0.297x + 46.$$

The "life expectancy from age 65" line contains the points $(0, 76)$ and $(110, 84.1)$.

$$m = \frac{84.1 - 76}{110 - 0} = \frac{8.1}{110} \approx 0.074$$

Since $(0, 76)$ is one of the points, the line is given by the equation

$$y = 0.074x + 76.$$

Set the two expressions for y equal to determine where the lines intersect. At this point, life expectancy should increase no further.

$$0.297x + 46 = 0.074x + 76$$
$$0.223x = 30$$
$$x \approx 135$$

Determine the y-value when $x = 129$. Use the first equation.

$$y = 0.297(135) + 46$$
$$= 40.095 + 46$$
$$= 86.095$$

Thus, the maximum life expectancy for humans is about 86 years.

70. (a) Let $t = 0$ correspond to 1900. Then the "mortality rate for children under 5 years of age" line contains the points $(90, 90)$ and $(112, 48)$.

$$m = \frac{48 - 90}{112 - 90} \approx -1.909$$

Use the point $(90, 90)$ and the point-slope form.

$$y - 90 = -1.909(t - 90)$$
$$y = -1.909t + 171.81 + 90$$
$$y = -1.909t + 261.81$$

(b) Let $y = 30$ and solve for t.

$$y = -1.909t + 261.81$$
$$30 = -1.909t + 261.81$$
$$-231.81 = -1.909t$$
$$121.43 = t$$

If the trend continues, the goal of the mortality rate for children under 5 years of age being 30 per 1000 live births would be reached in 2022.

71. (a) The line goes through $(50, 249{,}187)$ and $(112, 1{,}031{,}631)$.

$$m = \frac{1{,}031{,}631 - 249{,}187}{112 - 50} \approx 12{,}620.06$$

Use the point $(50, 249{,}187)$ and the point-slope form.

$$y - 249{,}187 = 12{,}620.06(t - 50)$$
$$y = 12{,}620.06t - 631{,}003 + 249{,}187$$
$$y = 12{,}620.06t - 381{,}816$$

(b) The year 2015 corresponds to $t = 115$.

$$y = 12{,}620.06(115) - 381{,}816$$
$$y \approx 1{,}069{,}491$$

The number of immigrants admitted to the United States in 2015 will be about 1,069,491.

(c) The equation $y = 12{,}620.16t - 381{,}816$ has $-381{,}816$ for the y-intercept, indicating that the number of immigrants admitted in the year 1900 was $-381{,}816$. Realistically, the number of immigrants cannot be a negative value, so the equation cannot be used for valid predicted values.

72. (a) The line (for the data for men) goes through $(0, 24.7)$ and $(30, 28.2)$.

$$m = \frac{28.2 - 24.7}{30 - 0} \approx 0.117$$

Use the point $(0, 24.7)$ and the point-slope form.

$$y - 24.7 = 0.117(t - 0)$$
$$y = 0.117t + 24.7$$

(b) The line (for the data for women) goes through $(0, 22.0)$ and $(30, 26.1)$.

$$m = \frac{26.1 - 22.0}{30 - 0} \approx 0.137$$

Use the point $(0, 22.0)$ and the point-slope form.

$$y - 22.0 = 0.137(t - 0)$$
$$y = 0.137t + 22.0$$

(c) Since $0.137 > 0.117$, women seem to have the faster increase in median age at first marriage.

(d) Let $y = 30$.

$$30 = 0.117t + 24.7$$
$$5.3 = 0.117t$$
$$45.299 \approx t$$

The median age at first marriage for men will reach 30 in the year $1980 + 45 = 2025$ or $1980 + 46 = 2026,$ depending on how the computations were rounded.

(e) Let $t = 45.299$.

$$y = 0.137(45.299) + 22.0$$
$$y \approx 28.2$$

The median age at first marriage for women will reach be 28.2 when the median age for men is 30. (The answer will be 28.3 if the year 2026 is used as the answer for part (d).)

73. (a) Plot the points $(15, 1600),\ (200, 15,000),$ $(290, 24,000),\ $ and $\ (520, 40,000).$
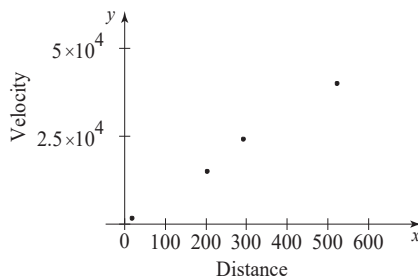
The points lie approximately on a line, so there appears to be a linear relationship between distance and time.

(b) The graph of any equation of the form $y = mx$ goes through the origin, so the line goes through $(520, 40,000)$ and $(0, 0).$

$$m = \frac{40,000 - 0}{520 - 0} \approx 76.9$$
$$b = 0$$
$$y = 76.9x + 0$$
$$y = 76.9x$$

(c) Let $y = 60,000;$ solve for $x.$

$$60,000 = 76.9x$$
$$780.23 \approx x$$

Hydra is about 780 megaparsecs from earth.

(d) $A = \dfrac{9.5 \times 10^{11}}{m},\ m = 76.9$

$$A = \frac{9.5 \times 10^{11}}{76.9}$$
$$= 12.4 \text{ billion years}$$

74. (a) If the temperature rises 0.3C° per decade, it rises 0.03C° per year.

$$m = 0.03$$

$b = 15,$ since a point is $(0,15).$

$$T = 0.03t + 15$$

(b) Let $T = 19;$ find $t.$

$$19 = 0.03t + 15$$
$$4 = 0.03t$$
$$t = 133.3 \approx 133$$

So, $1970 + 133 = 2103.$

The temperature will rise to 19°C in about the year 2103.

75. (a) Let $t = 0$ correspond to 2000. Then the line representing the percent of respondents who got their news from the newspaper contains the points (6, 40) and (12, 29).

$$m = \frac{29 - 40}{12 - 6} \approx -1.83$$

Use the point $(6, 40)$ and the point-slope form.

$$y_n - 40 = -1.83(t - 6)$$
$$y_n = -1.83t + 10.98 + 40$$
$$y_n = -1.83t + 50.98$$

(b) The line representing the percent of respondents who got their news online contains the points (6, 23) and (12, 39).

$$m = \frac{39 - 23}{12 - 6} \approx 2.67$$

Use the point $(6, 23)$ and the point-slope form.

$$y_o - 23 = 2.67(t - 6)$$
$$y_o = 2.67t - 16.02 + 23$$
$$y_o = 2.67t + 6.98$$

(c) The number of respondents who got their news from newspapers is decreasing by about 1.83% per year, while the number of respondents who got their news online is increasing by about 2.67% per year.

1.2   Linear Functions and Applications

Your Turn 1

For $g(x) = -4x + 5$, calculate $g(-5)$.

$$g(x) = -4x + 5$$
$$g(-5) = -4(-5) + 5$$
$$= 20 + 5$$
$$= 25$$

Your Turn 2

For the demand and supply functions given in Example 2, find the quantity of watermelon demanded and supplied at a price of \$3.30 per watermelon.

$$p = D(q) = 9 - 0.75q$$
$$3.30 = 9 - 0.75q$$
$$0.75q = 5.7$$
$$q = \frac{5.7}{0.75} = 7.6$$

Since the quantity is in thousands, 7600 watermelon are demanded at a price of \$3.30.

$$p = S(q) = 0.75q$$
$$3.30 = 0.75q$$
$$q = \frac{3.3}{0.75} = 4.4$$

Since the quantity is in thousands, 4400 watermelon are supplied at a price of \$3.30.

Your Turn 3

Set the two price expressions equal and solve for the equilibrium quantity q.

$$10 - 0.85q = 0.4q$$
$$10 = 1.25q$$
$$q = \frac{10}{1.25} = 8$$

The equilibrium quantity is 8000 watermelon. Use either price expression to find the equilibrium price p.

$$p = 0.4q$$
$$p = 0.4(8) = 3.2$$

The equilibrium price is \$3.20 per watermelon.

Your Turn 4

The marginal cost is the slope of the cost function $C(x)$, so this function has the form $C(x) = 15x + b$. To find b, use the fact that producing 80 batches costs \$1930.

$$C(x) = 15x + b$$
$$C(80) = 15(80) + b$$
$$1930 = 1200 + b$$
$$b = 730$$

Thus the cost function is $C(x) = 15x + 730$.

Your Turn 5

The fixed cost is b, this function has the form $C(x) = mx + 7145$. To find m, use the fact that producing 100 items costs \$7965.

$$C(x) = mx + 7145$$
$$C(100) = 100m + 7145$$
$$7965 = 100m + 7145$$
$$820 = 100m$$
$$m = 8.2$$

Thus the cost function is $C(x) = 8.2x + 7145$.

Your Turn 6

The cost function is $C(x) = 35x + 250$ and the revenue function is $R(x) = 58x$. Thus the profit function is

$$P(x) = R(x) - C(x)$$
$$= 58x - (35x + 250)$$
$$= 23x - 250$$

The profit is to be \$8030.

$$P(x) = 23x - 250$$
$$8030 = 23x - 250$$
$$23x = 8280$$
$$x = \frac{8280}{23} = 360$$

Sale of 360 units will produce \$8030 profit.

1.2   Warmup Exercises

W1. $3(x - 2)^2 + 6(x + 4) - 5x + 4$

$$3(5 - 2)^2 + 6(5 + 4) - 5(5) + 4$$
$$= 3(3)^2 + 6(9) - 5(5) + 4$$
$$= 27 + 54 - 25 + 4$$
$$= 60$$

W2.
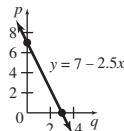
1.2   Exercises

1. $f(2) = 7 - 5(2) = 7 - 10 = -3$

2. $f(4) = 7 - 5(4) = 7 - 20 = -13$

3. $f(-3) = 7 - 5(-3) = 7 + 15 = 22$

4. $f(-1) = 7 - 5(-1) = 7 + 5 = 12$

5. $g(1.5) = 2(1.5) - 3 = 3 - 3 = 0$

6. $g(2.5) = (2.5) - 3 = 5 - 3 = 2$

7. $g\left(-\dfrac{1}{2}\right) = 2\left(-\dfrac{1}{2}\right) - 3 = -1 - 3 = -4$

8. $g\left(-\dfrac{3}{4}\right) = 2\left(-\dfrac{3}{4}\right) - 3 = -\dfrac{3}{2} - 3 = -\dfrac{9}{2}$

9. $f(t) = 7 - 5(t) = 7 - 5t$

10. $g(k^2) = 2(k^2) - 3 = 2k^2 - 3$

11. This statement is true.

When we solve $y = f(x) = 0$, we are finding the value of x when $y = 0$, which is the x-intercept. When we evaluate $f(0)$, we are finding the value of y when $x = 0$, which is the y-intercept.

12. This statement is false.

The graph of $f(x) = -5$ is a horizontal line.

13. This statement is true.

Only a vertical line has an undefined slope, but a vertical line is not the graph of a function. Therefore, the slope of a linear function cannot be undefined.

14. This statement is true.

For any value of a,
$$f(0) = a \cdot 0 = 0,$$
so the point $(0, 0)$, which is the origin, lies on the line.

15. The fixed cost is constant for a particular product and does not change as more items are made. The marginal cost is the rate of change of cost at a specific level of production and is equal to the slope of the cost function at that specific value; it approximates the cost of producing one additional item.

19. $10 is the fixed cost and $2.25 is the cost per hour.

Let $x = $ number of hours;

$R(x) = $ cost of renting a snowboard for x hours.

Thus,
$R(x) = $ fixed cost + (cost per hour) $\cdot$ (number of hours)
$R(x) = 10 + (2.25)(x)$
$\quad\ = 2.25x + 10$

20. $10 is the fixed cost and $0.99 is the cost per downloaded song—the marginal cost.

Let $x = $ the number of downloaded songs and $C(x) = $ cost of downloading x songs. Then,

$C(x) = $ (marginal cost) $\cdot$ (number of downloaded songs) + fixed cost

$C(x) = 0.99x + 10.$

21. $2 is the fixed cost and $0.75 is the cost per half-hour.

Let $x = $ the number of half-hours;

$C(x) = $ the cost of parking a car for x half-hours.

Thus,
$$C(x) = 2 + 0.75x$$
$$\qquad = 0.75x + 2$$

22. $44 is the fixed cost and $0.28 is the cost per mile.

Let $x = $ the number of miles;

$R(x) = $ the cost of renting for x miles.

Thus,
$R(x) = $ fixed cost + (cost per mile) $\cdot$ (number of miles)
$R(x) = 44 + 0.28x.$

23. Fixed cost, $100; 50 items cost $1600 to produce.

Let $C(x) = $ cost of producing x items.

$C(x) = mx + b$, where b is the fixed cost.
$$C(x) = mx + 100$$
Now,

$C(x) = 1600$ when $x = 50$, so
$$1600 = m(50) + 100$$
$$1500 = 50m$$
$$30 = m.$$
Thus, $C(x) = 30x + 100.$

24. Fixed cost: $35; 8 items cost $395.

Let $C(x) = $ cost of x items

$C(x) = mx + b$, where b is the fixed cost

$$C(x) = mx + 35$$
Now, $C(x) = 395$ when $x = 8$, so
$$395 = m(8) + 35$$
$$360 = 8m$$
$$45 = m.$$
Thus, $C(x) = 45x + 35.$

25. Marginal cost: $75; 50 items cost $4300.
$$C(x) = 75x + b$$

Now, $C(x) = 4300$ when $x = 50$.

$$4300 = 75(50) + b$$
$$4300 = 3750 + b$$
$$550 = b$$

Thus, $C(x) = 75x + 550$.

26. Marginal cost, \$120; 700 items cost \$96,500 to produce.

$$C(x) = 120x + b$$

Now, $C(x) = 96{,}500$ when $x = 700$.

$$96{,}500 = 120(700) + b$$
$$96{,}500 = 84{,}000 + b$$
$$12{,}500 = b$$

Thus, $C(x) = 120x + 12{,}500$.

27. $D(q) = 16 - 1.25q$

(a) $D(0) = 16 - 1.25(0) = 16 - 0 = 16$

When 0 watches are demanded, the price is \$16.

(b) $D(4) = 16 - 1.25(4) = 16 - 5 = 11$

When 400 watches are demanded, the price is \$11.

(c) $D(8) = 16 - 1.25(8) = 16 - 10 = 6$

When 800 watches are demanded, the price is \$6.

(d) Let $D(q) = 8$. Find q.

$$8 = 16 - 1.25q$$
$$\frac{5}{4}q = 8$$
$$q = 6.4$$

When the price is \$8, the number of watches demanded is 640.

(e) Let $D(q) = 10$. Find q.

$$10 = 16 - 1.25q$$
$$\frac{5}{4}q = 6$$
$$q = 4.8$$

When the price is \$10, the number of watches demanded is 480.

(f) Let $D(q) = 12$. Find q.

$$12 = 16 - 1.25q$$
$$\frac{5}{4}q = 4$$
$$q = 3.2$$

When the price is \$12, the number of watches demanded is 320.

(g)
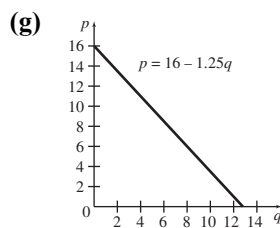
(h) $S(q) = 0.75q$

Let $S(q) = 0$. Find q.

$$0 = 0.75q$$
$$0 = q$$

When the price is \$0, the number of watches supplied is 0.

(i) Let $S(q) = 10$. Find q.

$$10 = 0.75q$$
$$\frac{40}{3} = q$$
$$q = 13.\overline{3}$$

When the price is \$10, The number of watches supplied is about 1333.

(j) Let $S(q) = 20$. Find q.

$$20 = 0.75q$$
$$\frac{80}{3} = q$$
$$q = 26.\overline{6}$$

When the price is \$20, the number of watches demanded is about 2667.

(k)
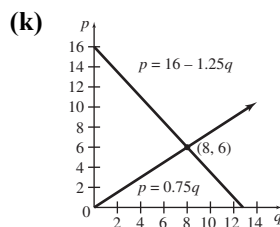
(l)
$$D(q) = S(q)$$
$$16 - 1.25q = 0.75q$$
$$16 = 2q$$
$$8 = q$$

$$S(8) = 0.75(8) = 6$$

The equilibrium quantity is 800 watches, and the equilibrium price is \$6.

28. $D(q) = 5 - 0.25q$

(a) $D(0) = 5 - 0.25(0) = 5 - 0 = 5$

When 0 quarts are demanded, the price is \$5.

(b) $D(4) = 5 - 0.25(4) = 5 - 1 = 4$

When 400 quarts are demanded, the price is $4.

(c) $D(8.4) = 5 - 0.25(8.4) = 5 - 2.1 = 2.9$

When 840 quarts are demanded, the price is $2.90.

(d) Let $D(q) = 4.5$. Find q.

$$4.5 = 5 - 0.25q$$
$$0.25q = 0.5$$
$$q = 2$$

When the price is $4.50, 200 quarts are demanded.

(e) Let $D(q) = 3.25$. Find q.

$$3.25 = 5 - 0.25q$$
$$0.25q = 1.75$$
$$q = 7$$

When the price is $3.25, 700 quarts are demanded.

(f) Let $D(q) = 2.4$. Find q.

$$2.4 = 5 - 0.25q$$
$$0.25q = 2.6$$
$$q = 10.4$$

When the price is $2.40, 1040 quarts are demanded.

(g)
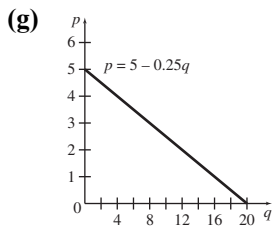
(h) $S(q) = 0.25q$

Let $S(q) = 0$. Find q.

$$0 = 0.25q$$
$$q = 0$$

When the price is $0, 0 quarts are supplied.

(i) Let $S(q) = 2$. Find q.

$$2 = 0.25q$$
$$q = 8$$

When the price is $2, 800 quarts are supplied.

(j) Let $S(q) = 4.5$. Find q.

$$4.5 = 0.25q$$
$$q = 18$$

When the price is $4.50, 1800 quarts are supplied.

(k)
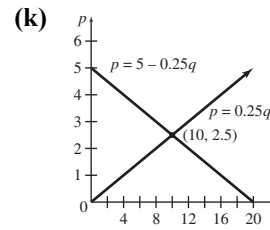
(l)
$$D(q) = S(q)$$
$$5 - 0.25q = 0.25q$$
$$5 = 0.5q$$
$$10 = q$$

$$S(10) = 0.25(10) = 2.5$$

The equilibrium quantity is 1000 quarts and the equilibrium price is $2.50.

29. $p = S(q) = \dfrac{2}{5}q;\ p = D(q) = 100 - \dfrac{2}{5}q$

(a)
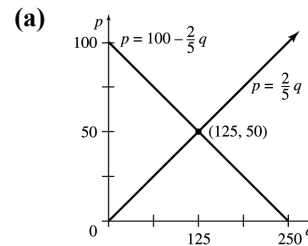
(b) $S(q) = D(q)$
$$\frac{2}{5}q = 100 - \frac{2}{5}q$$
$$\frac{4}{5}q = 100$$
$$q = 125$$
$$S(125) = \frac{2}{5}(125) = 50$$

The equilibrium quantity is 125, the equilibrium price is $50.

30. (a)
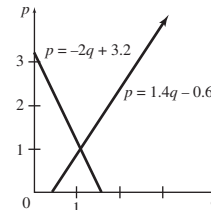
(b) $S(q) = p = 1.4q - 0.6$
$D(q) = p = -2q + 3.2$

Set supply equal to demand and solve for q.

$$1.4q - 0.6 = -2q + 3.2$$
$$1.4q + 2q = 0.6 + 3.2$$
$$3.4q = 3.8$$
$$q = \frac{3.8}{3.4} \approx 1.12$$
$$S(1.12) = 1.4(1.12) - 0.6 = 0.968$$

The equilibrium quantity is about 1120 pounds; the equilibrium price is about $0.96

31. Use the supply function to find the equilibrium quantity that corresponds to the given equilibrium price of $4.50.

$$S(q) = p = 0.3q + 2.7$$
$$4.50 = 0.3q + 2.7$$
$$1.8 = 0.3q$$
$$6 = q$$

The line that represents the demand function goes through the given point $(2, 6.10)$ and the equilibrium point $(6, 4.50)$.

$$m = \frac{4.50 - 6.10}{6 - 2} = -0.4$$

Use point-slope form and the point $(2, 6.10)$.

$$D(q) - 6.10 = -0.4(q - 2)$$
$$D(q) = -0.4q + 0.8 + 6.10$$
$$D(q) = -0.4q + 6.9$$

32. Use the supply function to find the equilibrium quantity that corresponds to the given equilibrium price of $5.85.

$$p = S(q)$$
$$5.85 = 0.25q + 3.6$$
$$2.25 = 0.25q$$
$$9 = q$$

The line that represents the demand function goes through the given point $(4, 7.60)$ and the equilibrium point $(9, 5.85)$.

$$m = \frac{5.85 - 7.60}{9 - 4}$$
$$= -0.35$$

Use point-slope form and the point $(4, 7.60)$.

$$D(q) - 7.60 = -0.35(q - 4)$$
$$D(q) = -0.35q + 1.4 + 7.60$$
$$D(q) = -0.35q + 9$$

33. $C(x) = 5x + 20; R(x) = 15x$

(a) $C(x) = R(x)$
$$5x + 20 = 15x$$
$$20 = 10x$$
$$2 = x$$

The break-even quantity is 2 units.

(b) $P(x) = R(x) - C(x)$
$$P(x) = 15x - (5x + 20)$$
$$P(100) = 15(100) - (5 \cdot 100 + 20)$$
$$= 1500 - 520$$
$$= 980$$

The profit from 100 units is $980.

(c) $P(x) = 500$
$$15x - (5x + 20) = 500$$
$$10x - 20 = 500$$
$$10x = 520$$
$$x = 52$$

For a profit of $500, 52 units must be produced.

34. $C(x) = 12x + 39; R(x) = 25x$

(a) $C(x) = R(x)$
$$12x + 39 = 25x$$
$$39 = 13x$$
$$3 = x$$

The break-even quantity is 3 units.

(b) $P(x) = R(x)$
$$P(x) = 25x - (12x + 39)$$
$$P(x) = 13x - 39$$
$$P(250) = 13(250) - 39$$
$$= 3250 - 39$$
$$= 3211$$

The profit from 250 units is $3211.

(c) $P(x) = \$130$; find x.
$$130 = 13x - 39$$
$$169 = 13x$$
$$13 = x$$

For a profit of $130, 13 units must be produced.

35. (a) $C(x) = mx + b; \ m = 3.50; \ C(60) = 300$
$$C(x) = 3.50x + b$$

Find b.
$$300 = 3.50(60) + b$$
$$300 = 210 + b$$
$$90 = b$$
$$C(x) = 3.50x + 90$$

(b) $R(x) = 9x$
$$C(x) = R(x)$$
$$3.50x + 90 = 9x$$
$$90 = 5.5x$$
$$16.36 = x$$

Joanne must produce and sell 17 shirts.

(c) $P(x) = R(x) - C(x); P(x) = 500$

$$500 = 9x - (3.50x + 90)$$
$$500 = 5.5x - 90$$
$$590 = 5.5x$$
$$107.27 = x$$

To make a profit of $500, Joanne must produce and sell 108 shirts.

36. (a) $C(x) = mx + b$

$$C(1000) = 2675; b = 525$$

Find m.

$$2675 = m(1000) + 525$$
$$2150 = 1000m$$
$$2.15 = m$$

$$C(x) = 2.15x + 525$$

(b) $R(x) = 4.95x$

$$C(x) = R(x)$$
$$2.15x + 525 = 4.95x$$
$$525 = 2.80x$$
$$187.5 = x$$

In order to break even, he must produce and sell 188 books.

(c) $P(x) = R(x) - C(x); P(x) = 1000$

$$1000 = 4.95x - (2.15x + 525)$$
$$1000 = 4.95x - 2.15x - 525$$
$$1000 = 2.80x - 525$$
$$1525 = 2.80x$$
$$544.6 = x$$

In order to make a profit of $1000, he must produce and sell 545 books.

37. (a) Using the points (100, 11.02) and (400, 40.12),

$$m = \frac{40.12 - 11.02}{400 - 100}$$
$$= \frac{29.1}{300} = 0.097.$$
$$y - 11.02 = 0.097(x - 100)$$
$$y - 11.02 = 0.097x - 9.7$$
$$y = 0.097x + 1.32$$
$$C(x) = 0.097x + 1.32$$

(b) The fixed cost is given by the constant in $C(x)$. It is $1.32.

(c) $C(1000) = 0.097(1000) + 1.32$

$$= 97 + 1.32$$
$$= 98.32$$

The total cost of producing 1000 cups is $98.32.

(d) $C(1001) = 0.097(1001) + 1.32$

$$= 97.097 + 1.32$$
$$= 98.417$$

The total cost of producing 10001 cups is $98.42.

(e) Marginal cost $= 98.417 - 98.32$

$$= \$0.097 \quad \text{or} \quad 9.7¢$$

(f) The marginal cost for *any* cup is the slope, $0.097 or 9.7¢. This means the cost of producing one additional cup of coffee would be 9.7¢.

38. $C(10,000) = 547,500; C(50,000) = 737,500$

(a) $C(x) = mx + b$

$$m = \frac{737,500 - 547,500}{50,000 - 10,000}$$
$$= \frac{190,000}{40,000}$$
$$= 4.75$$
$$y - 547,500 = 4.75(x - 10,000)$$
$$y - 547,500 = 4.75x - 47,500$$
$$y = 4.75x + 500,000$$
$$C(x) = 4.75x + 500,000$$

(b) The fixed cost is $500,000.

(c) $C(100,000) = 4.75(100,000) + 500,000$

$$= 475,000 + 500,000$$
$$= 975,000$$

The total cost to produce 100,000 items is $975,000.

(d) Since the slope of the cost function is 4.75, the marginal cost is $4.75. This means that the cost of producing one additional item at this production level is $4.75.

39. $C(x) = 85x + 900$

$R(x) = 105x$

Set $C(x) = R(x)$ to find the break-even quantity.

$$85x + 900 = 105x$$
$$900 = 20x$$
$$45 = x$$

The break-even quantity is 45 units. You should decide not to produce since no more than 38 units can be sold.

$$P(x) = R(x) - C(x)$$
$$= 105x - (85x + 900)$$
$$= 20x - 900$$

The profit function is $P(x) = 20x - 900$.

40. $C(x) = 105x + 6000$

$R(x) = 250x$

Set $C(x) = R(x)$ to find the break-even quantity.

$$105x + 6000 = 250x$$
$$6000 = 145x$$
$$41.38 \approx x$$

The break-even quantity is about 41 units, so you should decide to produce.

$$P(x) = R(x) - C(x)$$
$$= 250x - (105x + 6000)$$
$$= 145x - 6000$$

The profit function is $P(x) = 145x - 6000$.

41. $C(x) = 70x + 500$

$R(x) = 60x$

$$70x + 500 = 60x$$
$$10x = -500$$
$$x = -50$$

This represents a break-even quantity of -50 units. It is impossible to make a profit when the break-even quantity is negative. Cost will always be greater than revenue.

$$P(x) = R(x) - C(x)$$
$$= 60x - (70x + 500)$$
$$= -10x - 500$$

The profit function is $P(x) = -10x - 500$.

42. $C(x) = 1000x + 5000$

$R(x) = 900x$

$$900x = 1000x + 5000$$
$$-5000 = 100x$$
$$-50 = x$$

It is impossible to make a profit when the break-even quantity is negative. Cost will always be greater than revenue.

$$P(x) = R(x) - C(x)$$
$$= 900x - (1000x + 5000)$$
$$= -100x - 5000$$

The profit function is $P(x) = -100x - 5000$ (always a loss).

43. Since the fixed cost is $400, the cost function is $C(x) = mx + 100$, where m is the cost per unit.

The revenue function is $R(x) = px$, where p is the price per unit.

The profit $P(x) = R(x) - C(x)$ is 0 at the given break-even quantity of 80.

$$P(x) = px - (mx + 400)$$
$$P(x) = px - mx - 400$$
$$P(x) = Mx - 400 \qquad \text{(Let } M = p - m.)$$
$$P(80) = M \cdot 80 - 400$$
$$0 = 80M - 400$$
$$400 = 80M$$
$$5 = M$$

So, the linear profit function is $P(x) = 5x - 400$, and the marginal profit is 5.

44. Since the fixed cost is $650, the cost function is $C(x) = mx + 650$, where m is the cost per unit.

The revenue function is $R(x) = px$, where p is the price per unit.

The profit $P(x) = R(x) - C(x)$ is 0 at the given break-even quantity of 25.

$$P(x) = px - (mx + 650)$$
$$P(x) = px - mx - 650$$
$$P(x) = Mx - 650 \qquad \text{(Let } M = p - m.)$$
$$P(25) = M \cdot 25 - 650$$
$$0 = 25M - 650$$
$$650 = 25M$$
$$26 = M$$

So, the linear profit function is $P(x) = 26x - 650$, and the marginal profit is 26.

45. **(a)** $f(x) = 34x + 230$
$$1000 = 34x + 230$$
$$770 = 34x$$
$$x = 22.647$$

Approximately 23 acorns per square meter would produce 1000 deer tick larvae per 400 square meters.

(b) The slope is 34, which indicates that the number of deer tick larvae per 400 square meters in the spring will increase by 34 for each additional acorn per square meter in the fall.

46. **(a)** Let t correspond to the number of years since 1990. Then for the cause of death due to tobacco, we have at $t = 0$, $m = 2.8$ since the quantity was rising at the rate of 28 million years per decade and $b = 35$ since 35 million years of healthy life were lost. The linear function is $f_1(t) = 2.8t + 35$.

(b) For the cause of death due to diarrhea, we have at $t = 0$, $m = -2.2$ since the quantity was falling at the rate of 22 million years per decade and $b = 100$ since 100 million years of healthy life were lost. The linear function is $f_d(t) = -2.2t + 100$.

(c)
$$f_t(t) = f_d(t)$$
$$2.8t + 35 = -2.2t + 100$$
$$5.0t = 65$$
$$t = 13$$

The amount of healthy life lost to tobacco was expected to equal that lost to diarrhea in 2003.

47. Use the formula derived in Example 8 in this section of the textbook.
$$F = \frac{9}{5}C + 32 \quad \text{or} \quad C = \frac{5}{9}(F - 32)$$

(a) $F = 58$; find C.
$$C = \frac{5}{9}(58 - 32)$$
$$C = \frac{5}{9}(26) = 14.4$$
The temperature is 14.4°C.

(b) $F = -20$; find C.
$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}(-20 - 32)$$
$$C = \frac{5}{9}(-52) = -28.9$$
The temperature is -28.9°C.

(c) $C = 50$; find F.
$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(50) + 32$$
$$F = 90 + 32 = 122$$
The temperature is 122°F.

48. Use the formula derived in Example 8 in this section of the textbook.
$$F = \frac{9}{5}C + 32 \quad \text{or} \quad C = \frac{5}{9}(F - 32)$$

(a) $C = 37$; find F.

$$F = \frac{9}{5}(37) + 32$$
$$F = \frac{333}{5} + 32 = 98.6$$
The Fahrenheit equivalent of 37°C is 98.6°F.

(b) $C = 36.5$; find F.
$$F = \frac{9}{5}(36.5) + 32$$
$$F = 65.7 + 32 = 97.7$$

$C = 37.5$; find F.
$$F = \frac{9}{5}(37.5) + 32$$
$$= 67.5 + 32 = 99.5$$
The range is between 97.7°F and 99.5°F.

49. If the temperatures are numerically equal, then $F = C$.
$$F = \frac{9}{5}C + 32$$
$$C = \frac{9}{5}C + 32$$
$$-\frac{4}{5}C = 32$$
$$C = -40$$
The Celsius and Fahrenheit temperatures are numerically equal at $-40°$.

50. (a) $m = 1140$
$$b = 486,000$$
$$C(x) = mx + b$$
$$C(x) = 1140x + 486,000$$

(b) $C(500) = 1140 \cdot 500 + 486,000$
$$= 1,056,000$$
The total cost for 500 students will be $1,056,000.

(c) Let $C(x) = 1,000,000$.
$$1,000,000 = 1140x + 486,000$$
$$514,000 = 1140x$$
$$450.88 = x$$
The maximum number of students that each center can support for $1 million in costs is 450 students.

1.3   The Least Squares Line

Your Turn 1

x	y	xy	x^2	y^2
1	3	3	1	9
2	4	8	4	16
3	6	18	9	36
4	5	20	16	25
5	7	35	25	49
6	8	48	36	64

$\Sigma x = 21$ $\Sigma y = 33$ $\Sigma xy = 132$ $\Sigma x^2 = 91$ $\Sigma y^2 = 199$

The number of data points is $n = 6$. Putting the column totals into the formula for the slope m, we get

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$m = \frac{6(132) - (21)(33)}{6(91) - (21)^2}$$

$$m \approx 0.9429$$

$$b = \frac{\Sigma y - m(\Sigma x)}{n}$$

$$= \frac{33 - (0.9429)(21)}{6} \approx 2.2$$

The least square line is $Y = 0.9429x + 2.2$.

Your Turn 2
Put the column totals computed in Your Turn 1 into the formula for the correlation r.

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2} \cdot \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$= \frac{6(132) - (21)(33)}{\sqrt{6(91) - (21)^2} \cdot \sqrt{6(199) - (33)^2}}$$

$$\approx 0.9429$$

1.3   Exercises

3. (a)
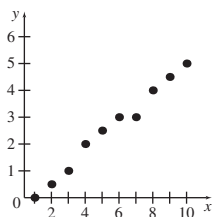
(b)

x	y	xy	x^2	y^2
1	0	0	1	0
2	0.5	1	4	0.25
3	1	3	9	1
4	2	8	16	4
5	2.5	12.5	25	6.25
6	3	18	36	9
7	3	21	49	9
8	4	32	64	16
9	4.5	40.5	81	20.25
10	5	50	100	25
55	25.5	186	385	90.75

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2} \cdot \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$= \frac{10(186) - (55)(25.5)}{\sqrt{10(385) - (55)^2} \cdot \sqrt{10(90.75) - (25.5)^2}}$$

$$\approx 0.993$$

(c)   The least squares line is of the form
$Y = mx + b$. First solve for m.

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$= \frac{10(186) - (55)(25.5)}{10(385) - (55)^2}$$

$$= 0.5545454545 \approx 0.555$$

Now find b.

$$b = \frac{\Sigma y - m(\Sigma x)}{n}$$

$$= \frac{25.5 - 0.5545454545(55)}{10}$$

$$= -0.5$$

Thus, $Y = 0.555x - 0.5$.
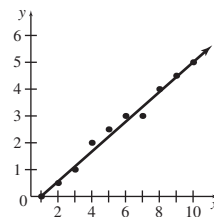
(d)   Let $x = 11$. Find Y.
$$Y = 0.55(11) - 0.5 = 5.6$$

4.

x	y	xy	x^2	y^2
6.8	0.8	5.44	46.24	0.64
7.0	1.2	8.4	49.0	1.44
7.1	0.9	6.39	50.41	0.81
7.2	0.9	6.48	51.84	0.81
7.4	1.5	11.1	54.76	2.25
35.5	5.3	37.81	252.25	5.95

$$r = \frac{5(37.81) - (35.5)(5.3)}{\sqrt{5(252.25) - (35.5)^2} \cdot \sqrt{5(5.95) - (5.3)^2}}$$

$$\approx 0.6985$$

$$r^2 = (0.6985)^2 \approx 0.5$$

The answer is choice (c).

5.

x	y	xy	x^2	y^2
1	1	1	1	1
1	2	2	1	4
2	1	2	4	1
2	2	4	4	4
9	9	81	81	81
15	15	90	91	91

(a) $n = 5$

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$= \frac{5(90) - (15)(15)}{5(91) - (15)^2}$$

$$= 0.9782608 \approx 0.9783$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$= \frac{15 - (0.9782608)(15)}{5} \approx 0.0652$$

Thus, $Y = 0.9783x + 0.0652$.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$$

$$= \frac{5(90) - (15)(15)}{\sqrt{5(91) - (15)^2} \cdot \sqrt{5(91) - (15)^2}} \approx 0.9783$$

(b)

x	y	xy	x^2	y^2
1	1	1	1	1
1	2	2	1	4
2	1	2	4	1
2	2	4	4	4
6	6	9	10	10

$n = 4$

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$= \frac{4(9) - (6)(6)}{4(10) - (6)^2} = 0$$

$$b = \frac{\sum y - m(\sum x)}{n} = \frac{6 - (0)(6)}{4} = 1.5$$

Thus, $Y = 0x + 1.5$, or $Y = 1.5$.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$$

$$= \frac{4(9) - (6)(6)}{\sqrt{4(10) - (6)^2} \cdot \sqrt{4(10) - (6)^2}}$$

$$= 0$$

(c)
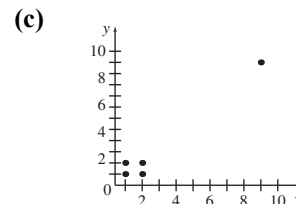
The point $(9, 9)$ is an outlier that has a strong effect on the least squares line and the correlation coefficient.

6.

x	y	xy	x^2	y^2
1	1	1	1	1
2	2	4	4	4
3	3	9	9	9
4	4	16	16	16
9	−20	−180	81	400
19	−10	−150	111	430

(a) $n = 5$

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(-150) - (19)(-10)}{5(111) - (19)^2}$$

$$= -2.886597 \approx -2.887$$

$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{-10 - (-2.886597)(19)}{5}$$

$$= 8.969069 \approx 8.969$$

Thus, $Y = -2.887x + 8.969$.

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{5(-150) - (19)(-10)}{\sqrt{5(111) - (19)^2} \cdot \sqrt{5(430) - (-10)^2}}$$

$$= -0.887994$$

$$\approx -0.8880$$

(b)

x	y	xy	x^2	y^2
1	1	1	1	1
2	2	4	4	4
3	3	9	9	9
4	4	16	16	16
10	10	30	30	30

$n = 4$

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{4(30) - (10)(10)}{4(30) - (10)^2} = 1$$

$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{10 - (1)(10)}{4} = 0$$

Thus, $Y = 1x + 0$, or $Y = x$.

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{4(30) - (10)(10)}{\sqrt{4(30) - (10)^2} \cdot \sqrt{4(30) - 10^2}}$$

$$= 1$$

(c)
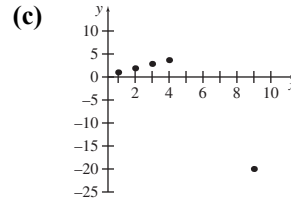
The point $(9, -20)$ is an outlier that has a strong effect on the least squares line and the correlation coefficient.

7.

x	y	xy	x^2	y^2
1	1	1	1	1
2	1	2	4	1
3	1	3	9	1
4	1.1	4.4	16	1.21
10	4.1	10.4	30	4.21

(a) $n = 4$

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{4(10.4) - (10)(4.1)}{\sqrt{4(30) - (10)^2} \cdot \sqrt{4(4.21) - (4.1)^2}}$$

$$= 0.7745966 \approx 0.7746$$

(b)
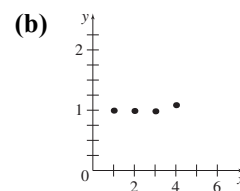
(c) Yes; because the data points are either on or very close to the horizontal line $y = 1$, it seems that the data should have a strong linear relationship. The correlation coefficient does not describe well a linear relationship if the data points fit a horizontal line.

8.

x	y	xy	x^2	y^2
0	4	0	0	16
1	1	1	1	1
2	0	0	4	0
3	1	3	9	1
4	4	16	16	16
10	10	20	30	34

(a) $n = 5$

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(20) - (10)(10)}{5(30) - (10)^2} = 0$$

$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{10 - (0)(10)}{5} = 2$$

Thus, $Y = 0x + 2$, or $Y = 2$.

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{5(20) - (10)(10)}{\sqrt{5(30) - (10)^2} \cdot \sqrt{5(34) - (10)^2}} = 0$$

(b)

(c) No; a correlation coefficient of 0 means that there isn't a linear relationship between the x and y values. A parabola (a quadratic relationship) seems to fit the given data points.
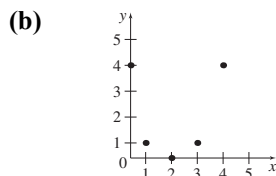
9. $nb + (\sum x)m = \sum y$

$(\sum x)b + (\sum x^2)m = \sum xy$

$nb + (\sum x)m = \sum y$

$nb = (\sum y) - (\sum x)m$

$b = \frac{\sum y - m(\sum x)}{n}$

$$(\sum x)\left(\frac{\sum y - m(\sum x)}{n}\right) + (\sum x^2)m = \sum xy$$

$(\sum x)[(\sum y) - m(\sum x)] + nm(\sum x^2) = n(\sum xy)$

$(\sum x)(\sum y) - m(\sum x)^2 + nm(\sum x^2) = n(\sum xy)$

$nm(\sum x^2) - m(\sum x)^2 = n(\sum xy) - (\sum x)(\sum y)$

$m[n(\sum x^2) - (\sum x)^2] = n(\sum xy) - (\sum x)(\sum y)$

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

10. (a)

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{10(338.271) - (75)(43.792)}{10(645) - (75)^2}$$

$$= 0.1191636 \approx 0.1192$$

$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{43.792 - (0.1192)(75)}{10}$$

$$= 3.4852 \approx 3.485$$

Thus, $Y = 0.1192x + 3.485$. .

(b) The year 2020 corresponds to $x = 20$.

$Y = 0.1192(20) + 3.485 = 5.869$

The total value of consumer durable goods in 2020 will be about \$5.869 trillion.

(c) Let $Y = 6$ and find x.

$6 = 0.1192x + 3.485$

$2.515 = 0.1192x$

$21.10 \approx x$

The total value of consumer durable goods will reach at least \$6 trillion in the year $2000 + 22 = 2022$.

(d)

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{10(338.271) - (75)(43.792)}{\sqrt{10(645) - (75)^2} \cdot \sqrt{10(193.0496) - (43.792)^2}}$$

$$\approx 0.9583$$

Since r is very close to 1, the data has a strong linear relationship, and the least squares line fits the data very well.

11. (a) $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{10(512.775) - (75)(70.457)}{10(645) - (75)^2}$

$= -0.189727 \approx 0.1897$

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{70.457 - (-0.1897)(75)}{10}$

$= 8.46845 \approx 8.469$

Thus, $Y = 00.1897x + 8.469$.

(b)   The year 2020 corresponds to $x = 20$.

$Y = -0.1897(20) + 8.469 = 4.675$

If the trend continues linearly, there will be about 4675 banks in 2020.

(c)   Let $Y = 4$ (since Y is the number of banks in thousands) and find x.

$4 = -0.1897x + 8.469$

$-4.469 = -0.1897x$

$23.558 = x$

$24 \approx x$

The number of U.S. banks will drop below 4000 in the year $2000 + 24 = 2024$.

(d) $r = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$

$= \dfrac{10(512.775) - (75)(70.457)}{\sqrt{10(645) - (75)^2} \cdot \sqrt{10(499.481335) - (70.457)^2}}$

≈ -0.9847

Since r is very close to -1 , the data has a strong linear relationship, and the least squares line fits the data very well.

12.

x	y	xy	x^2	y^2
0	41.5	0	0	1722.25
3	54.7	164.1	9	2992.09
7	61.7	431.9	49	3806.89
9	68.7	618.3	81	4719.69
12	74.8	897.6	144	5595.04
31	301.4	2111.9	283	18,835.96

(a) $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{5(2111.9) - (31)(301.4)}{5(283) - (31)^2}$

≈ 2.6786

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{301.4 - (2.6786)(31)}{5}$

≈ 43.67268

Thus, $Y = 2.68x + 43.7$.

(b)   The percent of households with Internet use is growing at a rate of about 2.68% per year.

(c)   The year 2015 corresponds to $x = 15$.

$Y = 2.68(15) + 43.7 = 83.9$

If the trend continues linearly, the percent of households will be about 83.9% in 2015.

(d)   Let $Y = 90$ and find x.

$90 = 2.68x + 43.7$

$46.3 = 2.68x$

$17.28 = x$

The percent of households with Internet use will exceed 90% in the year $2000 + 18 = 2018$.

(e) $r = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$

$= \dfrac{5(2111.9) - (31)(301.4)}{\sqrt{5(283) - (31)^2} \cdot \sqrt{5(18,835.96) - (301.4)^2}}$

≈ 0.9879

This means that the least squares line fits the data points extremely well.

13.

x	y	xy	x^2	y^2
5	89.7	448.5	25	8046.09
6	84.1	504.6	36	7072.81
7	81.9	573.3	49	6707.61
8	77.9	623.2	64	6068.41
9	73.5	661.5	81	5402.25
10	68.2	682.0	100	4651.24
11	63.8	701.8	121	4070.44
12	59.6	715.2	144	3552.16
68	598.7	4910.1	620	45,571.01

(a) $m = \dfrac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$

$= \dfrac{8(4910.1) - (68)(598.7)}{8(620) - (68)^2}$

≈ -4.2583

$b = \dfrac{\sum y - m\left(\sum x\right)}{n}$

$= \dfrac{598.7 - (-4.2583)(68)}{8}$

≈ 111.033

Thus, $Y = -4.26x + 111.0$.

(b) The percent of households with landlines is decreasing at a rate of about 4.26% per year.

(c) The year 2015 corresponds to $x = 15$.

$Y = -4.26(15) + 111.0 = 47.1$

If the trend continues linearly, the percent of households will be about 47.1% in 2012.

(d) Let $Y = 40$ and find x.

$$40 = -4.26x + 111.0$$
$$-71 = -4.26x$$
$$16.67 = x$$
$$17 \approx x$$

The percent of households with landlines will dip below 40% in the year $2000 + 17 = 2017$.

(e)

$r = \dfrac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$

$= \dfrac{8(4910.1) - (68)(598.7)}{\sqrt{8(620) - (68)^2} \cdot \sqrt{8(45,571.01) - (598.7)^2}}$

≈ -0.9973

This means that the least squares line fits the data points extremely well.

14.

x	y	xy	x^2	y^2
4	2219.5	8878	16	4,926,180.25
5	2319.8	11,599	25	5,381,472.04
6	2415.0	14,490	36	5,832,225
7	2551.9	17,863.3	49	6,512,193.61
8	2592.1	20,736.8	64	6,718,982.41
9	2553.5	22,981.5	81	6,520,362.25
10	2648.1	26,481	100	7,012,433.61
11	2757.0	30,327	121	7,601,049
12	2924.3	35,091.6	144	8,551,530.49
13	3099.2	40,289.6	169	9,605,040.64
85	26,080.4	228,737.8	805	68,661,469.3

(a) $m = \dfrac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$

$= \dfrac{10(228,737.8) - (85)(26,080.4)}{10(805) - (85)^2}$

≈ 85.508

$b = \dfrac{\sum y - m\left(\sum x\right)}{n}$

$= \dfrac{26,080.4 - (85.508)(85)}{10}$

≈ 1881.22

Thus, $Y = 85.508x + 1881.22$.

(b) The consumer credit is growing at a rate of about $85.508 billion per year.

(c) The year 2015 corresponds to $x = 15$.

$Y = 85.508(15) + 1881.22 = 3163.84$

If the trend continues linearly, the consumer credit will be about $3164 in 2015.

(d) Let $Y = 4000$ and find x.

$$4000 = 85.508x + 1881.22$$
$$2118.78 = 85.508x$$
$$24.78 = x$$

The total debt will first   exceed $4000 billion in the year $2000 + 25 = 2025$.

(e)

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{10(228,737.8) - (85)(26,080.4)}{\sqrt{10(805) - (85)^2} \cdot \sqrt{10(68,661,469.3) - (26,080.4)^2}}$$

$$\approx 0.9688$$

This means that the least squares line fits the data points extremely well.

15. (a)
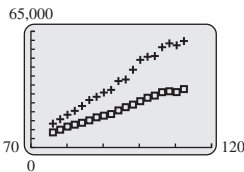
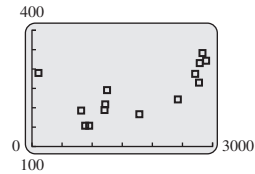The data points lie in a linear pattern.

(b) $Y = 703.91x - 45,220$.

$r = 0.9964$; there is a strong positive correlation among the data.

(c) The mean earnings of high school graduates are growing by about $704 per year.

(d) $Y = 1404.76x - 94,521$.

$r = 0.9943$; there is a strong positive correlation among the data.

(e) The mean earning of workers with a bachelor's degree are growing by about $1405 per year.

(f)
$$75,000 = 703.91x - 45,220$$
$$120,220 = 703.91x$$
$$x = 170.79$$

The mean earnings for high school graduates will exceed $75,000 in the year $1900 + 171 = 2071$.

$$75,000 = 1404.76x - 94,521$$
$$1169,521 = 1404.76x$$
$$x = 120.68$$

The mean earning for workers with a bachelor's degree will exceed $75,00 in the year $1900 + 121 = 2021$.
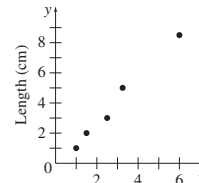
16. (a)

The data points lie in a linear pattern.

(b) $r = 6.210$; there is a positive correlation among the data. Yes, the cost of a ticket tends to increase as the distance flown increases.

(c) $Y = -0.417x + 167.55$; the marginal cost per mile is about 4.17 cents per mile.

(d) The outlier in the scatterplot is Philadelphia.

17. (a)

Yes, the data appear to be linear.

(b)

x	y	xy	x^2	y^2
5.8	8.6	49.88	33.64	73.96
1.5	1.9	2.85	2.25	3.61
2.3	3.1	7.13	5.29	9.61
1.0	1.0	1.0	1.0	1.0
3.3	5.0	16.5	10.89	25.0
13.9	19.6	77.36	53.07	113.18

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$= \frac{5(77.36) - (13.9)(19.6)}{5(53.07) - 13.9^2}$$

$$= 1.585250901 \approx 1.585$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$= \frac{19.6 - 1.585250901(13.9)}{5} \approx -0.487$$

$$Y = 1.585x - 0.487$$
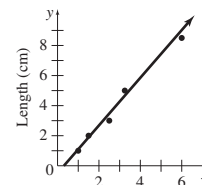
(c) No, it gives negative values for small widths.

(d) $r = \dfrac{5(77.36) - (13.9)(19.6)}{\sqrt{5(53.07) - 13.9^2} \cdot \sqrt{5(113.18) - 19.6^2}}$

≈ 0.999

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{249.8 - 0.211925(1200.6)}{15} \approx -0.309$

$Y = 0.212x - 0.309$

(b) Let $x = 73$; find Y.

$Y = 0.212(73) - 0.309 \approx 15.2$
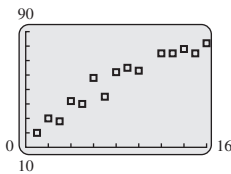
If the temperature were 73° F, you would expect to hear 15.2 chirps per second.

(c) Let $Y = 18$; find x.

$18 = 0.212x - 0.309$

$18.309 = 0.212x$

$86.4 \approx x$

When the crickets are chirping 18 times per second, the temperature is 86.4°F.
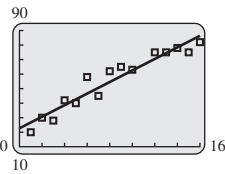
18. (a)

Yes, the points lie in a linear pattern.

(b) Using a calculator's STAT feature, the correlation coefficient is found to be $r \approx 0.959$.

This indicates that the percentage of successful hunts does trend to increase with the size of the hunting party.

(c) $Y = 3.98x + 22.7$

(d) $r = \dfrac{15(20,127) - (1200.6)(249.8)}{\sqrt{15(96,725.86) - (1200.6)^2} \cdot \sqrt{15(4200.56) - (249.8)^2}}$

$= 0.835$

19. (a)

x	y	xy	x^2	y^2
88.6	20.0	1772	7849.96	400.0
71.6	16.0	1145.6	5126.56	256.0
93.3	19.8	1847.34	8704.89	392.04
84.3	18.4	1551.12	7106.49	338.56
80.6	17.1	1378.26	6496.36	292.41
75.2	15.5	1165.6	5655.04	240.25
69.7	14.7	1024.59	4858.09	216.09
82.0	17.1	1402.2	6724	292.41
69.4	15.4	1068.76	4816.36	237.16
83.3	16.2	1349.46	3938.89	262.44
79.6	15.0	1194	6336.16	225
82.6	17.2	1420.72	6822.76	295.84
80.6	16.0	1289.6	6496.36	256.0
83.5	17.0	1419.5	6972.25	289.0
76.3	14.4	1098.72	5821.69	207.36
1200.6	249.8	20,127.47	96,725.86	4200.56

$m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{15(20,127.47) - (1200.6)(249.8)}{15(96,725.86) - 1200.6^2}$

$= 0.211925009 \approx 0.212$

20.

x	y	xy	x^2	y^2
0	17.2	0	0	295.84
4	17.3	69.2	16	299.29
8	16.4	131.2	64	268.96
12	15.9	190.8	144	252.81
16	15.6	249.6	256	243.36
20	15.3	306	400	234.09
60	97.7	946.8	880	1594.35

(a) $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{6(946.8) - (60)(97.7)}{6(880) - (60)^2}$

≈ -0.1079

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{97.7 - (-0.1079)(60)}{6}$

≈ 17.36

Thus, $Y = -0.1079x + 1736$.

(b) The year 2020 corresponds to
$x = 2020 - 1990 = 30.$

$Y = -0.1079(30) + 17.36 = 14.123 \approx 14.1$

If the trend continues linearly, the pupil-teacher
ratio will be about 14.1 in 2020.

(c) $r = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$
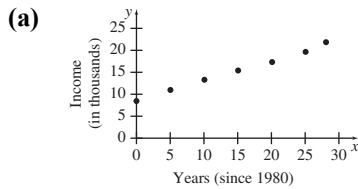
$= \dfrac{6(946.8) - (60)(97.7)}{\sqrt{6(880) - (60)^2} \cdot \sqrt{6(1594.35) - (97.7)^2}}$

≈ -0.9691

The value indicates a strong negative linear
correlation.

21.

x	y	xy	x^2	y^2
0	8.414	0.000	0	70.7954
5	10.989	54.945	25	120.7581
10	13.359	133.590	100	178.4629
15	15.569	233.535	225	242.3938
20	17.604	352.080	400	309.9008
25	19.971	499.275	625	398.8408
30	22.315	669.45	900	497.9592
105	108.221	1942.875	2275	1819.111

(a)

Yes, the data appear to lie along a straight line.

(b)

$r = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$

$= \dfrac{7(1942.875) - (105)(108.221)}{\sqrt{7(2275) - (105)^2} \cdot \sqrt{7(1819.111) - (108.221)^2}}$

$= 0.9996$

The value indicates a strong linear correlation.

(c)

$m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{7(1942.85) - (105)(108.221)}{7(2275) - (105)^2} \approx 0.45651 \approx 0.4565$

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{108.221 - (0.45651)(105)}{7} \approx 8.612$

Thus, $Y = 0.4565x + 8.612.$

(d) The year 2018 corresponds to $x = 2018 - 1980 = 38.$

$Y = 0.45651(38) + 8.612 = 25.959$

The predicted poverty level in the year 2018
is \$25,959.

22.

x	y	xy	x^2	y^2
59	66	3894	3481	4356
62	71	4402	3844	5041
66	72	4752	4356	5184
68	73	4964	4624	5329
71	75	5325	5041	5625
67	63	4221	4489	3969
70	63	4410	4900	3969
71	67	4757	5041	4489
73	66	4818	5329	4356
75	66	4950	5625	4356
682	682	46,493	46,730	46,674

(a) $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$

$= \dfrac{10(46,493) - (682)(682)}{10(46,730) - (682)^2} \approx -0.08915$

$b = \dfrac{\sum y - m(\sum x)}{n}$

$= \dfrac{682 - (-0.08915)(682)}{10} \approx 74.28$

Thus, $Y = -0.08915x + 74.28.$

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{10(46,493) - (682)(682)}{\sqrt{10(46,730) - (682)^2} \cdot \sqrt{10(46,674) - (682)^2}}$$

$$\approx -0.1035$$

The taller the student, the shorter the ideal partner's height is.

(b)   Data for female students:

x	y	xy	x^2	y^2
59	66	3894	3481	4356
62	71	4402	3844	5041
66	72	4752	4356	5184
68	73	4964	4624	5329
71	75	5325	5041	5625
326	357	23,337	21,346	25,535

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(23,337) - (326)(357)}{5(21,346) - (326)^2} \approx 0.6674$$

$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{357 - (0.6674)(326)}{5} \approx 27.89$$

Thus, $Y = 0.6674x + 27.89$.

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{5(23,337) - (326)(357)}{\sqrt{5(21,346) - (326)^2} \cdot \sqrt{5(25,535) - (357)^2}}$$

$$\approx 0.9459$$

Data for male students:

x	y	xy	x^2	y^2
67	63	4221	4489	3969
70	63	4419	4900	3969
71	67	4757	5041	4489
73	66	4818	5329	4356
75	66	4950	5625	4356
356	325	23,156	25,384	21,139

$$m = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$$= \frac{5(23,156) - (356)(325)}{5(25,384) - (356)^2} \approx 0.4348$$

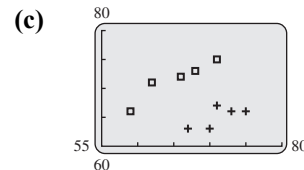$$b = \frac{\sum y - m\left(\sum x\right)}{n}$$

$$= \frac{325 - (0.4348)(356)}{5} \approx 34.04$$

Thus, $Y = 0.4348x + 34.04$.

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

$$= \frac{5(23,156) - (356)(325)}{\sqrt{5(25,384) - (356)^2} \cdot \sqrt{5(21,139) - (325)^2}}$$

$$\approx 0.7049$$

(c)

There is no linear relationship among all 10 data pairs. However, there is a linear relationship among the first five data pairs (female students) and a separate linear relationship among the second five data pairs (male students).

23.   (a)

x	y	xy	x^2	y^2
540	20	10,800	291,600	400
510	16	8160	260,100	256
490	10	4900	240,100	100
560	8	4480	313,600	64
470	12	5640	220,900	144
600	11	6600	360,000	121
540	10	5400	291,600	100
580	8	4640	336,400	64
680	15	10,200	462,400	225
560	8	4480	313,600	64
560	13	7280	313,600	169
500	14	7000	250,000	196
470	10	4700	220,900	100
440	10	4400	193,600	100
520	11	5720	270,400	121
620	11	6820	384,400	121

680	8	5440	462,400	64
550	8	4400	302,500	64
620	7	4340	384,400	49
10,490	210	115,400	5,872,500	2522

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{19(115,400) - (10,490)(210)}{19(5,872,500) - 10,490^2}$$

$$m = -0.0066996227 \approx -0.0067$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$b = \frac{210 - (-0.0066996227)(10,490)}{19} \approx 14.75$$

$$Y = -0.0067x + 14.75$$

(b) Let $x = 420$; find Y.

$$Y = -0.0067(420) + 14.75$$
$$= 11.936 \approx 12$$

(c) Let $x = 620$; find Y.

$$Y = -0.0067(620) + 14.75$$
$$= 10.596 \approx 11$$

(d)

$$r = \frac{19(115,400) - (10,490)(210)}{\sqrt{19(5,872,500) - (10,490)^2} \cdot \sqrt{19(2522) - 210^2}}$$

$$\approx -0.13$$

(e) There is no linear relationship between a student's math SAT and mathematics placement test scores.

24. (a)
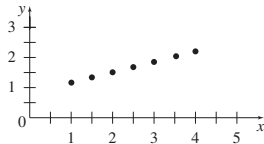
(b)

L	T	LT	L^2	T^2
1.0	1.11	1.11	1	1.2321
1.5	1.36	2.04	2.25	1.8496
2.0	1.57	3.14	4	2.4649
2.5	1.76	4.4	6.25	3.0976
3.0	1.92	5.76	9	3.6864
3.5	2.08	7.28	12.25	4.3264
4.0	2.22	8.88	16	4.9844
17.5	12.02	32.61	50.75	21.5854

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{7(32.61) - (17.5)(12.02)}{7(50.75) - 17.5^2}$$

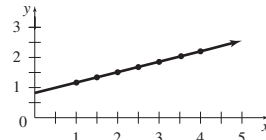$$m = 0.3657142857$$
$$\approx 0.366$$

$$b = \frac{\sum T - m(\sum L)}{n}$$

$$b = \frac{12.02 - 0.3657142857(17.5)}{7}$$

$$\approx 0.803$$

$$Y = 0.366x + 0.803$$

The line seems to fit the data.

(c)

$$r = \frac{7(32.61) - (17.5)(12.02)}{\sqrt{7(50.75) - 17.5^2} \cdot \sqrt{7(21.5854) - 12.02^2}}$$

$$= 0.995,$$

which is a good fit and confirms the conclusion in part (b).

25. (a)

x	y	xy	x^2	y^2
150	5000	750,000	22,500	25,000,000
175	5500	962,500	30,625	30,250,000
215	6000	1,290,000	46,225	36,000,000
250	6500	1,625,000	62,500	42,250,000
280	7000	1,960,000	78,400	49,000,000
310	7500	2,325,000	96,100	56,250,000
350	8000	2,800,000	122,500	64,000,000
370	8500	3,145,000	136,900	72,250,000
420	9000	3,780,000	176,400	81,000,000
450	9500	4,275,000	202,500	90,250,000
2970	72,500	22,912,500	974,650	546,250,000

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{10(22,912,500) - (2970)(72,500)}{10(974,650) - 2970^2}$$

$$m = 14.90924806$$

$$\approx 14.9$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$b = \frac{72,500 - 14.9(2970)}{10}$$

$$b \approx 2820$$

$$Y = 14.9x + 2820$$

(b) Let $x = 150$; find Y.

$$Y = 14.9(150) + 2820$$

$$Y \approx 5060, \text{ compared to actual } 5000$$

Let $x = 280$; Find Y.

$$Y = 14.9(280) + 2820$$

$$\approx 6990, \text{ compared to actual } 7000$$

Let $x = 420$; find Y.

$$Y = 14.9(420) + 2820$$

$$\approx 9080, \text{ compared to actual } 9000$$

(c) Let $x = 230$; find Y.

$$Y = 14.9(230) + 2820$$

$$\approx 6250$$

Adam would need to buy a 6500 BTU air conditioner.

26. (a)

$$r = \frac{10(399.16) - (500)(20.668)}{\sqrt{10(33,250) - 500^2} \cdot \sqrt{10(91.927042) - (20.668)^2}}$$

$$= -0.995$$

Yes, there does appear to be a linear correlation.

(b) $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{10(399.16) - (500)(20.668)}{10(33,250) - 500^2}$$

$$m = -0.0768775758 \approx -0.0769$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$b = \frac{20.668 - (-0.0768775758)(500)}{10}$$

$$\approx 5.91$$

$$Y = -0.0769x + 5.91$$

(c) Let $x = 50$

$$Y = -0.0769(50) + 5.91 \approx 2.07$$

The predicted number of points expected when a team is at the 50 yard line is 2.07 points.

27. (a) Use a calculator's statistical features to obtain the least squares line.

$$Y = -0.1271x + 113.61$$

(b) $Y = -0.3458x + 146.65$

(c) Set the two expressions for Y equal and solve for x.

$$-0.1271x + 113.61 = -0.3458x + 146.65$$
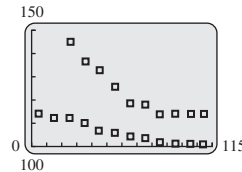
$$0.2187x = 33.04$$

$$x \approx 151$$

The women's record will catch up with the men's record in $1900 + 151$, or in the year 2051.

(d) $$r_{men} \approx -0.9762$$

$$r_{women} \approx -0.9300$$

Both sets of data points closely fit a line with negative slope.

(e)

28. (a) Use a calculator's statistical features to obtain the least squares line.

$$Y = -0.01116x + 10.92$$

(b) $Y = -0.01544x + 12.31$

(c) Set the two expressions for Y equal and solve for x.

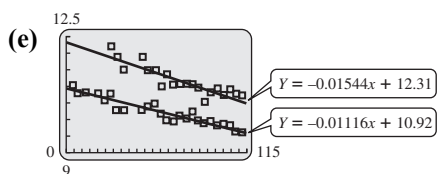$$-0.01116x + 10.92 = -0.01544x + 12.31$$

$$0.00428x = 1.39$$

$$x \approx 324.77$$

The women's record will catch up with the men's record in $1900 + 325$, or in the year 2225.

(d) $$r_{men} \approx -0.9506$$
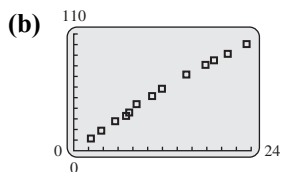
$$r_{women} \approx -0.8896$$

Both sets of data points closely fit a line with negative slope.

(e)

29.

x	y
0	0.0
2.317	11.5
3.72	18.9
5.6	27.8
7.08	32.8
7.5	36.0
8.5	43.9
10.6	51.5
11.93	58.4
15.23	71.8
17.82	80.9
18.97	85.2
20.83	91.3
23.38	100.5

(a) Skaggs' average speed was
$100.5/23.38 \approx 4.299$ miles per hour.

(b)

The data appear to lie approximately on a
straight line.

(c) Using a graphing calculator,

$$Y = 4.317x + 3.419.$$

(d) Using a graphing calculator,

$$r \approx 0.9971$$

Yes, the least squares line is a very good fit
to the data.

(e) A good value for Skaggs' average speed
would be the slope of the least squares line, or

$$m = 4.317 \text{ miles per hour.}$$

This value is faster than the average speed
found in part (a). The value 4.317 miles per
hour is most likely the better value because it
takes into account all 14 data pairs.

Chapter 1 Review Exercises

1. False; a line can have only one slant, so its slope is unique.

2. False; the equation $y = 3x + 4$ has slope 3.

3. True; the point $(3, -1)$ is on the line because
$-1 = -2(3) + 5$ is a true statement.

4. False; the points $(2, 3)$ and $(2, 5)$ do not have the
same y-coordinate.

5. True; the points $(4, 6)$ and $(5, 6)$ do have the same
y-coordinate.

6. False; the x-intercept of the line $y = 8x + 9$ is
$-\frac{9}{8}$.

7. True; $f(x) = \pi x + 4$ is a linear function because
it is in the form $y = mx + b$, where m and b are
real numbers.

8. False; $f(x) = 2x^2 + 3$ is not linear function
because it isn't in the form $y = mx + b$, and it is
a second-degree equation.

9. False; the line $y = 3x + 17$ has slope 3, and the
line $y = -3x + 8$ has slope -3. Since
$3 \cdot -3 \neq -1$, the lines cannot be perpendicular.

10. False; the line $4x + 3y = 8$ has slope $-\frac{4}{3}$, and
the line $4x + y = 5$ has slope -4. Since the
slopes are not equal, the lines cannot be parallel.

11. False; a correlation coefficient of zero indicates
that there is no linear relationship among the data.

12. True; a correlation coefficient always will be a
value between -1 and 1.

13. Marginal cost is the rate of change of the cost
function; the fixed cost is the initial expenses
before production begins.

14. To compute the coefficient of correlation, you
need the following quantities:
$\sum x, \sum y, \sum xy, \sum x^2, \sum y^2$, and n.

15. Through $(-3, 7)$ and $(2, 12)$

$$m = \frac{12 - 7}{2 - (-3)} = \frac{5}{5} = 1$$

16. Through $(4, -1)$ and $(3, -3)$.

$$m = \frac{-3 - (-1)}{3 - 4}$$
$$= \frac{-3 + 1}{-1}$$
$$= \frac{-2}{-1} = 2$$

17. Through the origin and $(11, -2)$

$$m = \frac{-2 - 0}{11 - 0} = -\frac{2}{11}$$

18. Through the origin and $(0, 7)$

$$m = \frac{7 - 0}{0 - 0} = \frac{7}{0}$$

The slope of the line is undefined.

19. $4x + 3y = 6$

$$3y = -4x + 6$$
$$y = -\frac{4}{3}x + 2$$

Therefore, the slope is $m = -\frac{4}{3}$.

20. $4x - y = 7$

$$-y = -4x + 7$$
$$y = 4x - 7$$
$$m = 4$$

21. $y + 4 = 9$

$$y = 5$$
$$y = 0x + 5$$
$$m = 0$$

22. $3y - 1 = 14$

$$3y = 14 + 1$$
$$3y = 15$$
$$y = 5$$

This is a horizontal line. The slope of a horizontal line is 0.

23. $y = 5x + 4$

$$m = 5$$

24. $x = 5y$

$$\frac{1}{5}x = y$$
$$m = \frac{1}{5}$$

25. Through $(5, -1)$; slope $\frac{2}{3}$

Use point-slope form.

$$y - (-1) = \frac{2}{3}(x - 5)$$
$$y + 1 = \frac{2}{3}(x - 5)$$
$$3(y + 1) = 2(x - 5)$$
$$3y + 3 = 2x - 10$$
$$3y = 2x - 13$$
$$y = \frac{2}{3}x - \frac{13}{3}$$

26. Through $(8, 0)$, with slope $-\frac{1}{4}$

Use point-slope form.

$$y - 0 = -\frac{1}{4}(x - 8)$$
$$y = -\frac{1}{4}x + 2$$

27. Through $(-6, 3)$ and $(2, -5)$

$$m = \frac{-5 - 3}{2 - (-6)} = \frac{-8}{8} = -1$$

Use point-slope form.

$$y - 3 = -1[x - (-6)]$$
$$y - 3 = -x - 6$$
$$y = -x - 3$$

28. Through $(2, -3)$ and $(-3, 4)$

$$m = \frac{4 - (-3)}{-3 - 2} = -\frac{7}{5}$$

Use point-slope form.

$$y - (-3) = -\frac{7}{5}(x - 2)$$
$$y + 3 = -\frac{7}{5}x + \frac{14}{5}$$
$$y = -\frac{7}{5}x + \frac{14}{5} - 3$$
$$y = -\frac{7}{5}x + \frac{14}{5} - \frac{15}{5}$$
$$y = -\frac{7}{5}x - \frac{1}{5}$$

29. Through $(2, -10)$, perpendicular to a line with undefined slope

A line with undefined slope is a vertical line. A line perpendicular to a vertical line is a horizontal line with equation of the form $y = k$. The desired line passed through $(2, -10)$, so $k = -10$. Thus, an equation of the desired line is $y = -10$.

30. Through $(-2, 5)$, with slope 0

Horizontal lines have 0 slope and an equation of the form $y = k$.

The line passes through $(-2, 5)$ so $k = 5$. An equation of the line is $y = 5$.

31. Through $(3, -4)$ parallel to $4x - 2y = 9$

Solve $4x - 2y = 9$ for y.

$$-2y = -4x + 9$$
$$y = 2x - \frac{9}{2}$$
$$m = 2$$

The desired line has the same slope. Use the point-slope form.

$$y - (-4) = 2(x - 3)$$
$$y + 4 = 2x - 6$$
$$y = 2x - 10$$

Rearrange.

$$2x - y = 10$$

32. Through $(0, 5)$, perpendicular to $8x + 5y = 3$

Find the slope of the given line first.

$$8x + 5y = 3$$
$$5y = -8x + 3$$
$$y = \frac{-8}{5}x + \frac{3}{5}$$
$$m = -\frac{8}{5}$$

The perpendicular line has $m = \frac{5}{8}$.

Use point-slope form.

$$y - 5 = \frac{5}{8}(x - 0)$$
$$y = \frac{5}{8}x + 5$$

Rearrange.

$$8y = 5x + 40$$
$$5x - 8y = -40$$

33. Through $(-1, 4)$; undefined slope

Undefined slope means the line is vertical.

The equation of the vertical line through $(-1, 4)$ is $x = -1$.

34. Through $(7, -6)$, parallel to a line with undefined slope.

A line with undefined slope has the form $x = a$ (a vertical line). The vertical line that goes through $(7, -6)$ is the line $x = 7$.

35. Through $(3, -5)$, parallel to $y = 4$

Find the slope of the given line.

$y = 0x + 4$, so $m = 0$, and the required line will also have slope 0.

Use the point-slope from.

$$y - (-5) = 0(x - 3)$$
$$y + 5 = 0$$
$$y = -5$$

36. Through $(-3, 5)$, perpendicular to $y = -2$ The given line, $y = -2$, is a horizontal line. A line perpendicular to a horizontal line is a vertical line with equation of the form $x = h$.

The desired line passes through $(-3, 5)$, so $h = -3$. Thus, an equation of the desired line is $x = -3$.

37. $y = 4x + 3$
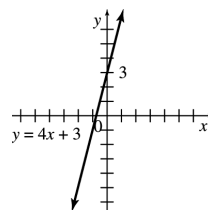
Let $x = 0$: $y = 4(0) + 3$
$$y = 3$$

Let $y = 0$: $0 = 4x + 3$
$$-3 = 4x$$
$$-\frac{3}{4} = x$$

Draw the line through $(0, 3)$ and $\left(-\frac{3}{4}, 0\right)$.

38. $y = 6 - 2x$

Find the intercepts.

Let $x = 0$.
$$y = 6 - 2(0) = 6$$
The y-intercept is 6.

Let $y = 0$.
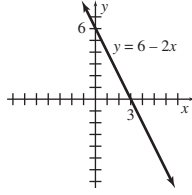$$0 = 6 - 2x$$
$$2x = 6$$
$$x = 3$$
The x-intercept is 3.

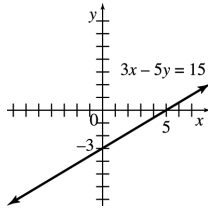Draw the line through (0, 6) and (3, 0).

39. $3x - 5y = 15$

$-5y = -3x + 15$

$y = \dfrac{3}{5}x - 3$

When $x = 0, y = -3$.

When $y = 0, x = 5$.

Draw the line through $(0, -3)$ and $(5, 0)$.

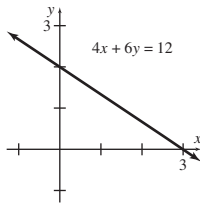**40.** $4x + 6y = 12$

Find the intercepts.

When $x = 0, y = 2$, so the y-intercept is 2.
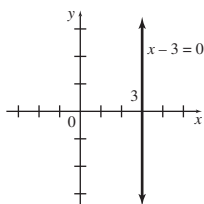
When $y = 0, x = 3$, so the x-intercept is 3.

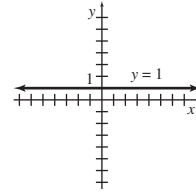Draw the line through $(0, 2)$ and $(3, 0)$.

41. $x - 3 = 0$

$x = 3$

This is the vertical line through $(3, 0)$.

42. $y = 1$

This is the horizontal line passing through $(0, 1)$.

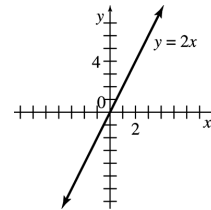**43.** $y = 2x$

When $x = 0, y = 0$.

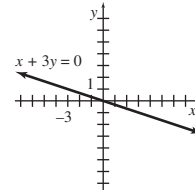When $x = 1, y = 2$.

Draw the line through $(0, 0)$ and $(1, 2)$.

44. $x + 3y = 0$

When $x = 0, y = 0$.

When $x = 3, y = -1$.

Draw the line through $(0, 0)$ and $(3, -1)$.

45. **(a)** Let $t = 0$ represent the year 2000. The line goes through (0, 100) and (13, 440).

$$m = \frac{440 - 100}{13 - 0}$$

$$= \frac{340}{13} \approx 26.2$$

$$y - 100 = 26.2(t - 0)$$

$$y = 26.2t + 100$$

(b) The imports from China are increasing by about \$26.2 billion per year.

(c) Let $t = 15$.

$$y = 26.2t + 100$$

$$y = 26.2(15) + 100$$

$$y = 493$$

The amount of imports from China in 2015 will be about \$493 billion.

(d) Let $y = 600$; solve for t.

$$600 = 26.2t + 100$$
$$500 = 26.2t$$
$$t \approx 19.084$$

The imports from China would be at least $600 billion in the year $2000 + 20 = 2020$.

46. **(a)** Let $t = 0$ represent the year 2000. The line goes through (0, 16) and (13, 122).

$$m = \frac{122 - 16}{13 - 0}$$
$$= \frac{106}{13} \approx 8.15$$
$$y - 16 = 8.15(t - 0)$$
$$y = 8.15t + 16$$

(b) The exports to China are increasing by about $8.15 billion per year.

(c) Let $t = 15$.

$$y = 8.15t + 16$$
$$y = 8.15(15) + 16$$
$$y = 138.25$$

The amount of exports to China in 2015 will be about $138 billion.

(d) Let $y = 200$; solve for t.

$$200 = 8.15t + 16$$
$$184 = 8.15t$$
$$t \approx 22.577$$

The exports to China would be at least $200 billion in the year $2000 + 23 = 2023$.

47. **(a)** Let $t = 0$ represent the year 2000. The line goes through (7, 55,627) and (12, 51,017).

$$m = \frac{51,017 - 55,627}{12 - 7}$$
$$= \frac{-4610}{5} \approx -922$$
$$y - 55,627 = -922(t - 7)$$
$$y = -922t + 6454 + 55,627$$
$$y = -922t + 62,081$$

(b) The median income for all U.S. households is decreasing by about $922 per year.

(c) Let $t = 15$.

$$y = -922t + 62,081$$
$$y = -922(15) + 62,081$$
$$y = 48,251$$

The median income for all U.S. households in 2015 will be about $48,251.

(d) Let $y = 40,000$; solve for x.

$$40,000 = -922t + 62,081$$
$$-22,081 = -922t$$
$$t \approx 23.949$$

The median income would drop below $40,000 in the year $2000 + 24 = 2024$.

48. $p = S(q) - 4q + 10; p = D(q) = 40 - 4q$

(a) $20 = 4q + 10$ $20 = 40 - 4q$

 $10 = 4q$ $-20 = -4q$

 $\dfrac{5}{2} = q$ (supply) $5 = q$ (demand)

When the price is $20 per pound, the supply is 2.5 pounds per day, and the demand is 5 pounds per day.

(b) $24 = 4q + 10$ $24 = 40 - 4q$

 $14 = 4q$ $-16 = -4q$

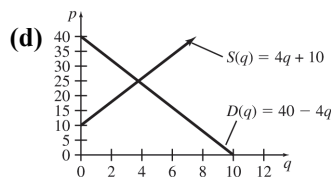$$ $\dfrac{7}{2} = q$ (supply) $4 = q$ (demand)

When the price is $24 per pound, the supply is 3.5 pounds per day, and the demand is 4 pounds per day.

(c) $32 = 4q + 10$ $32 = 40 - 4q$

 $22 = 4q$ $-8 = -4q$

 $\dfrac{11}{2} = q$ (supply) $2 = q$ (demand)

When the price is $32 per pound, the supply is 5.5 pounds per day, and the demand is 2 pounds per day.

(d)

(e) The graph shows that the lines representing the supply and demand functions intersect at the point (3.75, 25). The y-coordinate of this point gives the equilibrium price. Thus, the equilibrium price is $25 per pound.

(f) The x-coordinate of the intersection point gives the equilibrium quantity. Thus, the equilibrium quantity is 3.75, representing 3.75 pounds of crabmeat per day.

49. **(a)** The line that represents the supply function goes through the points (60, 40) and (100, 60).

$$m = \frac{60 - 40}{100 - 60} \approx 0.5$$

Use $(60, 40)$ and the point-slope form.

$$p - 40 = 0.5(q - 60)$$
$$p = 0.5q - 30 + 40$$
$$p = S(q) = 0.5q + 10$$

(b) The line that represents the demand function goes through the points $(50, 47.50)$ and $(80, 32.50)$.

$$m = \frac{32.50 - 47.50}{80 - 50} \approx -0.5$$

Use $(50, 47.50)$ and the point-slope form.

$$p - 47.50 = -0.5(q - 50)$$
$$p = 0.5q + 25 + 47.50$$
$$p = D(q) = -0.5q + 72.50$$

(c) Set supply equal to demand and solve for q.

$$0.5q + 10 = -0.5q + 72.50$$
$$1q = 62.50$$
$$q = 62.5$$
$$S(62.5) = 0.5(62.5) + 10 = 41.25$$

The equilibrium quantity is about 62.5 dietary supplement pills, and the equilibrium price is about $41.25.

50. Eight units cost $300; fixed cost is $60.
The fixed cost is the cost if zero units are made.
$(8, 300)$ and $(0, 60)$ are points on the line.

$$m = \frac{60 - 300}{0 - 8} = 30$$

Use slope-intercept form.

$$y = 30x + 60$$
$$C(x) = 30x + 60$$

51. Fixed cost is $2000; 36 units cost $8480.

Two points on the line are $(0, 2000)$ and $(36, 8480)$, so

$$m = \frac{8480 - 2000}{36 - 0} = \frac{6480}{36} = 180.$$

Use point-slope form.

$$y = 180x + 2000$$
$$C(x) = 180x + 2000$$

52. Twelve units cost $445; 50 units cost $1585.
Points on the line are $(12, 445)$ and $(50, 1585)$.

$$m = \frac{1585 - 445}{50 - 12} = 30$$

Use point-slope form.

$$y - 445 = 30(x - 12)$$
$$y - 445 = 30x - 360$$
$$y = 30x + 85$$
$$C(x) = 30x + 85$$

53. Thirty units cost $1500; 120 units cost $5640. Two points on the line are $(30, 1500)$, $(120, 5640)$, so

$$m = \frac{5640 - 1500}{120 - 30} = \frac{4140}{90} = 46.$$

Use point-slope form.

$$y - 1500 = 46(x - 30)$$
$$y = 46x - 1380 + 1500$$
$$y = 46x + 120$$
$$C(x) = 46x + 120$$

54. $C(x) = 200x + 1000$
$R(x) = 400x$

(a) $C(x) = R(x)$

$$200x + 1000 = 400x$$
$$1000 = 200x$$
$$5 = x$$

The break-even quantity is 5 cartons.

(b) $R(5) = 400(5) = 2000$

The revenue from 5 cartons of CD's is $2000.

55. (a) $C(x) = 3x + 160; R(x) = 7x$

$$C(x) = R(x)$$
$$3x + 160 = 7x$$
$$160 = 4x$$
$$40x = x$$

The break-even quantity is 40 pounds.

(b) $R(40) = 7 \cdot 40 = \$280$

The revenue for 40 pounds is $280.

56. (a) $E(x) = 42x + 352$ (where x is in thousands)
(b) $R(x) = 130x$ (where x is in thousands)
(c) $R(x) > E(x)$

$$130x > 42x + 352$$
$$88x > 352$$
$$x > 4$$

For a profit to be made, more than 4000 chips must be sold.

57.

x	y	x	y
2	26,150	8	28,350
3	27,550	9	28,966
4	28,050	10	29,793
5	28,400	11	30,659
6	28,450	12	30,910
7	28,200		

(a) Use the points $(2, 26{,}150)$ and $(12, 30{,}910)$ to find the slope.

$$m = \frac{30{,}910 - 26{,}150}{12 - 2} = 476$$

$$y - 26{,}150 = 476(t - 2)$$
$$y - 26{,}150 = 476t - 952$$
$$y = 476t + 25{,}198$$

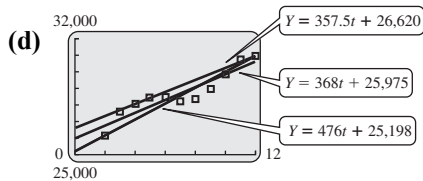(b) Use the points $(4, 28{,}050)$ and $(12, 30{,}910)$ to find the slope.

$$m = \frac{30{,}910 - 28{,}050}{12 - 4} = 357.5$$

$$y - 28{,}050 = 357.5(t - 4)$$
$$y - 28{,}050 = 357.5t - 1430$$
$$y = 357.5t + 26{,}620$$

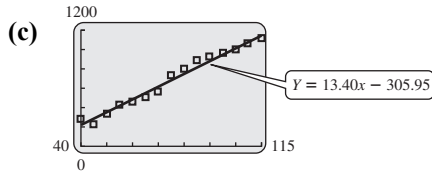(c) Using a graphing calculator, the least squares line is $Y = 386t + 25{,}975$.

(d)

(e) $r = 0.9356$

58. (a) $Y = 13.40x - 305.95$

(b) $Y = 13.40(115) - 305.95$
$Y \approx \$1235$

(c)

The data points lie in a linear pattern.

(d) $r = 0.9898$; There is a strong positive correlation among the data.

59.

t	Y (Beef)	y (Pork)	y (Chicken)
0	64.5	47.8	54.2
12	54.5	42.6	56.6

(a) For $b(t)$: Use the points $(0, 64.5)$ and $(12, 54.4)$ to find the slope.

$$m = \frac{54.5 - 64.5}{12 - 0} = -0.833$$

$$y - 64.5 = -0.833(t - 0)$$
$$y = -0.833t + 64.5$$

For $p(t)$: Use the points $(0, 47.8)$ and $(12, 42.6)$ to find the slope.

$$m = \frac{42.6 - 47.8}{12 - 0} = -0.433$$
$$y - 47.8 = -0.433(t - 0)$$
$$y = -0.433t + 47.8$$

For $c(t)$: Use the points $(0, 54.2)$ and $(12, 56.6)$ o find the slope.

$$m = \frac{56.6 - 54.2}{12 - 0} = 0.2$$
$$y - 54.2 = 0.2(t - 0)$$
$$y = 0.2t + 54.2$$

(b) Beef is decreasing by about 0.833 lb/yr; pork is decreasing by about 0.433 lb/yr; chicken is increasing by about 0.2 lb/yr.

(c) $-0.833t + 64.5 = 0.2t + 54.2$
$$-1.033t = -10.3$$
$$t \approx 9.97$$

The consumption of chicken surpassed the consumption of beef in the year
$2000 + 10 = 2010$.

(d) $y = -0.833t + 64.5$
$y = -0.833(15) + 64.5 \approx 52.005$

$y = -0.433t + 47.8$
$y = -0.433(15) + 47.8 \approx 41.305$
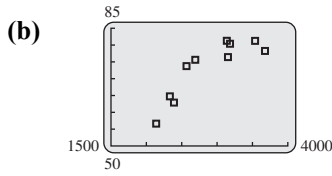
$y = 0.2t + 54.2$
$y = 0.2(15) + 54.2 = 57.2$

The consumption of beef will be about 52.0 lb, the consumption of pork will be about 41.3 lb, and the consumption of chicken will be about 57.2 lb in 2015.

60.

x	y
2680	75.6
2382	62.7
3531	81.2
2321	64.7
3146	76.5
3172	80.4
2563	73.6
3125	81.1
2137	56.6
3668	78.2

(a) Using a graphing calculator, $r \approx 0.8664$.
Yes, the data seem to fit a straight line.

(b)

(c) Using a graphing calculator,
$Y = 0.01423x + 32.19$. The data
somewhat fit a straight, but a curve would fit
the data better.

(d) Let $x = 3399$. Find Y.

$Y = 0.01423(3399) + 32.19 \approx 80.56$

The predicted life expectancy in Canada,
with a daily calorie supply of 3399, is about
80.6 years. This agrees with the actual value
of 80.8 years.

(e) The higher daily calorie supply most likely
contains more healthy nutrients, which might
result in a longer life expectancy.

61. (a)

x	y	xy	x^2	y^2
130	170	22,100	16,900	28,900
138	160	22,080	19,044	25,600
142	173	24,566	20,164	29,929
159	181	28,779	25,281	32,761
165	201	33,165	27,225	40,401
200	192	38,400	40,000	36,864
210	240	50,400	44,100	57,600
250	290	72,500	62,500	84,100
1394	1607	291,990	255,214	336,155

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{8(291,990) - (1394)(1607)}{8(225,214) - 1394^2}$$

$$m = 0.9724399854 \approx 0.9724$$

$$b = \frac{\sum y - m(\sum x)}{n}$$

$$b = \frac{1607 - 0.9724(1394)}{8} \approx 31.43$$

$$Y = 0.9724x + 31.43$$

(b) Let $x = 190$; find Y.

$$Y = 0.9724(190) + 31.43$$
$$Y = 216.19 \approx 216$$

The cholesterol level for a person whose
blood sugar level is 190 would be about 216.

(c) $r = \dfrac{8(291,990) - (1394)(1607)}{\sqrt{8(255,214) - 1394^2} \cdot \sqrt{8(336,155) - 1607^2}}$

$= 0.933814 \approx 0.93$

62.

t	y (Males)	y (Females)
5	31.0	23.5
23	34.4	28.6

(a) For $m(t)$: Use the points $(5, 31.0)$ and
$(23, 34.4)$ to find the slope.

$$m = \frac{34.4 - 31.0}{23 - 5} = 0.189$$
$$y - 31.0 = 0.189(t - 5)$$
$$y = 0.189t - 0.945 + 31.0$$
$$y = 0.189t + 30.055$$

(b) The percent of never-married males is
increasing by about 0.189 percent per year.

(c) For $f(t)$: Use the points $(5, 23.5)$ and
$(23, 28.6)$ to find the slope.

$$m = \frac{28.6 - 23.5}{23 - 5} = 0.283$$
$$y - 23.5 = 0.283(t - 5)$$
$$y = 0.283t - 1.415 + 23.5$$
$$y = 0.283t + 22.085$$

(d) The percent of never-married females is
increasing by about 0.283 percent per year.

(e) $y = 0.189t + 30.055$

$y = 0.189(25) + 30.055 \approx 34.78$

$y = 0.283t + 22.085$

$y = 0.283(25) + 22.085 \approx 29.16$

The percent of never-married males will be
about 34.78%, and the percent of never-
married females will be about 29.16% in 2015.

63. (a) Use the points $(0, 6400)$ and $(12, 9520)$ to
find the slope.

$$m = \frac{9520 - 6400}{12 - 0} = 260$$

$$b = 6400$$

The linear equation for the number of
families below the poverty level since 2000 is
$y = 260t + 6400$.

(b) Use the points $(4, 7835)$ and $(12, 9520)$ to find the slope.

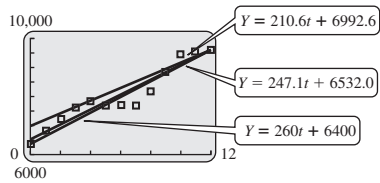$$m = \frac{9520 - 7835}{12 - 4} = 210.6$$

$$y - 7835 = 210.6(t - 4)$$
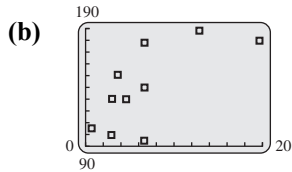
$$y - 7835 = 210.6t - 842.4$$

$$y = 210.6t + 6992.6$$

The linear equation for the number of families below the poverty level since 2000 is $y = 210.6t + 6992.6$.

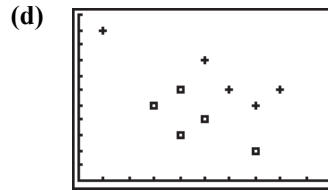**(c)** Using a graphing calculator, the least squares line is $Y = 247.1t + 6532.0$.

(d) The least squares line best describes the data. Since the data seems to fit a straight line, a linear model describes the data well.

(e) Using a graphing calculator, $r \approx 0.9515$.

64. (a) Using a graphing calculator, $r = 0.6889$. The data seem to fit a line but the fit is not very good.

(b)

(c) Using a graphing calculator,
$$Y = 4.156x + 111.5$$

(d) The slope is 4.156 thousand (or 4156). On average, the governor's salary increases $4156 for each additional million in population.

65. Use a graphing calculator to find these correlations.

(a) Correlation between years since 2000 and length: $r = 0.4529$

(b) Correlation between length and rating: $r = 0.3955$

(c) Correlation between years since 2000 and rating: $r = -0.4768$

(d)

This calculator graph plots year (on the horizontal axis) versus rating (on the vertical axis). Squares represent movies with lengths no more than 110 minutes, and plus signs represent movies with lengths 115 minutes or more.

Extended Application: Using Extrapolation to Predict Life Expectancy

1.

x	y	xy	x^2	y^2
1970	74.7	147,159.0	3,880,900	5580.09
1975	76.6	151,285.0	3,900,625	5867.56
1980	77.4	153,252.0	3,920,400	5990.76
1985	78.2	155,227.0	3,940,225	6115.24
1990	78.8	156,812.0	3,960,100	6209.44
1995	78.9	157,405.5	3,980,025	6225.21
2000	79.3	158,600.0	4,000,000	6288.49
2005	80.1	160,600.5	4,020,025	6416.01
2010	81.0	162,810.0	4,040,100	6561.0
17,910	705.0	1,403,151	35,642,400	55,253.8

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$m = \frac{9(1,403,151) - (17,910)(705.0)}{9(35,642,400) - 17,910^2} \approx 0.1340$$
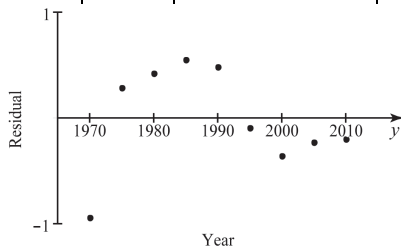
$$b = \frac{\sum y - m(\sum x)}{n}$$

$$b = \frac{705.0 - 0.1340(17,910)}{9} \approx -188.3267$$

$$Y = 0.134x - 188.33$$

2. Let $x = 1900$. Find Y.
$$Y = 0.134(1900) - 188.33 \approx 66.27$$

From the equation, the guess for the life expectancy of females born in 1900 is 66.27 years.

3. The poor prediction isn't surprising, since we were extrapolating far beyond the range of the original data.

5.

x	y	Predicted value	Residual
1970	74.7	75.65	−0.95
1975	76.6	76.32	0.28
1980	77.4	76.99	0.41
1985	78.2	77.66	0.54
1990	78.8	78.33	0.47
1995	78.9	79.0	− 0.1
2000	79.3	79.67	− 0.37
2005	80.1	80.34	− 0.24
2010	81.0	81.01	− 0.01

6. You'll get 0 slope and 0 intercept, because you've already subtracted out the linear component of the data.

7. A cubic would fit the data;

$$Y = 0.0002202x^3 - 1.3165x^2 + 2623.68x - 1,742,891.3$$